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On locally finite groups whose cyclic subgroups are monopronormal

The description of locally finite groups whose cyclic subgroups are monopronormal was obtained.

Key words: monopronormal subgroup, locally finite group, locally nilpotent residual.

Було отримано опис локально скінченних груп, циклічні підгрупи яких монопронормальні.

Ключові слова: монопронормальна підгрупа, локально скінченна група, локально нільпотентний резидуал.

Было получено описание локально конечных групп, циклические подгруппы которых монопронормальны.

Ключевые слова: монопронормальная подгруппа, локально конечная группа, локально нильпотентный резидуал.

1. Introduction

The investigation of influence of some systems of subgroups on the group structure is one of the oldest problem in group theory. For example, normal subgroups have a very strong influence on the group structure. Nevertheless, there are another important subgroups that have a significant effect on the group structure. We have in mind some antipodes of normal subgroups and their generalizations (for example, abnormal subgroups, self-normalizing subgroups, contranormal subgroups and others). Recall that a subgroup H of a group G is said to be *abnormal* in G if for each element $g \in G$ we have $g \in \langle H, H^g \rangle$. Recall also that a subgroup H of a group G is *contranormal* in G if $H^G = G$. Note that every abnormal subgroup is contranormal (see, for example, [1]).

On the other hand, there are subgroups that combine the concepts of normal subgroups and abnormal subgroups. One of the examples of such subgroups are pronormal subgroups. A subgroup H of a group G is said to be *pronormal* in G if for each element $g \in G$ the subgroups H and H^g are conjugate in $\langle H, H^g \rangle$. Note the following property of pronormal subgroups. If H is pronormal in G, then $N_G(H)$ is an abnormal subgroup of G (see, for example, [2]), and hence contranormal in G.

The first paper, devoted to the study of the influence of certain systems of subgroups on the group structure, is a classical article of R. Dedekind [3], in which he described the structure of finite groups whose all subgroups are normal. Later this result was extended to the case of infinite groups [4]. A group G is called a *Dedekind group*, if its all subgroups are normal. By [4], if G is a Dedekind group, then G is either abelian or $G = Q \times D \times B$, where Q is a quaternion group, D is an elementary abelian 2-subgroup and B is a periodic abelian $2'$-subgroup.

Let $\mathcal{P}$ and $\mathcal{AP}$ are subgroup properties. Moreover, suppose that all $\mathcal{AP}$-subgroups are antipodes (in some sense) to all $\mathcal{P}$-subgroups. There are many papers devoted to the study of the structure of groups whose subgroups either $\mathcal{P}$-subgroups or $\mathcal{AP}$-subgroups. In the present paper we consider the local (in some sense) version of this situation. Taking into account the above remarks on abnormal, contranormal and pronormal subgroups, we naturally obtain the following concept.

Definition 1. Let G be a group and H be a subgroup of G. A subgroup H is called *monopronormal* in G if for every element $x \in G$ either $H^x = H$ or $N_K(H)^K = K$, where $K = \langle H, x\rangle$, $x \in G$.

In the paper [5], it has been obtained the description of locally finite groups whose all subgroups are monopronormal. The next step here is to consider the locally finite groups whose cyclic subgroups are monopronormal.

2. Preliminary results

Lemma 1. *Let G be a group whose cyclic subgroups are monopronormal.*

(i) If H is a subgroup of G, then every cyclic subgroup of H is monopronormal.

(ii) If H is a normal subgroup of G, then every cyclic subgroup of G/H is monopronormal.

Proof. It follows from the definition of monopronormal subgroups.

Lemma 2. *Let G be a group and H be an ascendant subgroup of G. If H is a monopronormal subgroup of G, then H is normal in G.*

Proof. Let $H = H_0 \trianglelefteq H_1 \trianglelefteq \ldots H_\alpha \trianglelefteq H_{\alpha+1} \ldots H_\gamma = G$ be an ascending series between H and G. We will prove that H is normal in each H_α for all $\alpha \leq \gamma$. We will use a transfinite induction.

Let $\alpha = 1$. Then we have $H = H_0 \trianglelefteq H_1 = G$, which implies that $H \trianglelefteq G$. Assume now that H is normal in H_β for all $\beta < \alpha$. If α is a limit ordinal, then $H_\alpha = \bigcup\limits_{\beta<\alpha} H_\beta$. Let x be an arbitrary element of H_α. Then $x \in H_\beta$ for some $\beta < \alpha$. By induction hypothesis, H is normal in H_β. This means that $H^x = H$ for each $x \in H_\alpha$, which implies that $H \trianglelefteq H_\alpha$.

Suppose now that α is not a limit ordinal. Let x be an arbitrary element of H_α. If $H^x = H$, then $H \trianglelefteq H_\alpha$. Suppose that $H^x \neq H$. Put $K = \langle H, x\rangle$. By induction hypothesis, H is a (proper) normal subgroup of $H_{\alpha-1}$. Then H^x is a subgroup of $H_{\alpha-1}$. Moreover, H^x is normal in $H_{\alpha-1}$, which implies that $K = HH^x\langle x\rangle$. Since $H \leq N_K(H)$

and $H^x \leq N_K(H)$, $HH^x \leq N_K(H)$. In other words, $N_K(H) = HH^x\langle y\rangle$ for some $y \in \langle x\rangle$. If we suppose that $y = x$, then H is normal in K, and we obtain a contradiction with the condition $H^x \neq H$. Thus, $y \neq x$, which implies, that $N_K(H)$ is normal in K. This means, that $N_K(H)^K \neq K$.

Thus, we have equality $H^x = H$ for every $x \in H_\alpha$, which implies that $H \trianglelefteq H_\alpha$. For $\alpha = \gamma$ we obtain that H is normal in $H_\gamma = G$.

Corollary 1. *Let G be a group and H be a subnormal subgroup of G. If H is monopronormal in G, then H is normal in G.*

Corollary 2. *Let G be a nilpotent group. If every cyclic subgroup of G is monopronormal in G, then every subgroup of G is normal in G.*

Proof. It follows from the fact that every subgroup of a nilpotent group is subnormal.

Corollary 3. *Let G be a nilpotent group. If every cyclic subgroup of G is monopronormal in G, then G is a Dedekind group.*

Proof. It follows from Corollary 2 and the description of Dedekind groups.

Corollary 4. *Let G be a locally nilpotent group. If every cyclic subgroup of G is monopronormal in G, then G is a Dedekind group.*

Proof. Let x, y are arbitrary elements of G. Put $K = \langle x, y\rangle$. Then K is a nilpotent subgroup of G. Since $\langle x\rangle$ is monopronormal in K, by Corollary 2, $\langle x\rangle$ is normal in K. Therefore, $\langle x\rangle^y = \langle x\rangle$. This is valid for every element $y \in G$, which implies that $\langle x\rangle$ is normal in G. Since every cyclic subgroup of G is normal in G, then every subgroup of G is normal in G. Thus, G is a Dedekind group.

Lemma 3. *Let G be a group and K be a finite subgroup of G. Suppose that every cyclic subgroup of G is monopronormal in G. Let p be the least prime of $\Pi(K)$. Then $K = R \leftthreetimes P$ where P (respectively R) is a Sylow p-subgroup (respectively p'-subgroup) of K.*

Proof. Let P be a Sylow p-subgroup of K. By Corollary 3, P is a Dedekind group. Put $T = N_K(P)$. Then every cyclic subgroup of P is subnormal in T, and by Corollary 1, it is normal in T. It follows that P has a T-chief series whose factors have order p. Let U, V are T-invariant subgroups of P such that $U \leq V$ and V/U is a T-chief factor. By the proven above, $|V/U| = p$. Then $|T/C_T(V/U)|$ divides $p - 1$, and the choice of p implies that $T = C_T(V/U)$. In other words, every T-chief factor of P is central in T. Hence, P has a T-central series. It follows that $T = P \times S$ where S is a Sylow p'-subgroup of T. Suppose first that $p = 2$. Using now [6, Theorem 1] we obtain a following semidirect decomposition $K = R \leftthreetimes P$ where R is a Sylow $2'$-subgroup of K. If $p \neq 2$ then the description of Dedekind groups shows, that P is abelian. Using now a Burnside's theorem (see, for example, [7, Theorem 10.21]), we obtain that $K = R \leftthreetimes P$ where R is a Sylow p'-subgroup of K.

Corollary 5. *Let G be a group, K be a finite subgroup of G. If every cyclic subgroup of G is monopronormal in G, then K is soluble.*

Proof. Let D be a Sylow 2-subgroup of K. By Lemma 3, we have $K = R \leftthreetimes D$, where R is a Sylow $2'$-subgroup of K. The subgroup R is soluble [8], therefore K is also soluble.

Lemma 4. *Let G be a group, K be a finite subgroup of G. If every cyclic subgroup of G is monopronormal in G, then K has a Sylow tower.*

Proof. Let $\Pi(K) = \{p_1, \ldots, p_k\}$ and suppose that $p_1 < p_2 < \ldots < p_k$. We will prove this assertion using induction in $|\Pi(K)| = k$. If $k = 1$, then K is a p_1-subgroup, and all is proved. Assume now that $k > 1$. By Lemma 3, $K = R \leftthreetimes P$, where P is a Sylow p_1-subgroup of K and R is a normal Sylow p_1'-subgroup of K. We have now $\Pi(R) = \{p_2, \ldots, p_k\}$, and therefore by the induction hypothesis R has a Sylow tower. Since R is normal in K, every term of this Sylow tower is K-invariant, which proves the assertion.

Corollary 6. *Let G be a group, K be a finite subgroup of G. If every cyclic subgroup of G is monopronormal in G, then K is supersoluble.*

Proof. Let $\Pi(K) = \{p_1, \ldots, p_k\}$ and suppose that $p_1 < p_2 < \ldots < p_k$. By Lemma 4, K has a series of normal subgroups $K = S_0 > S_1 > \ldots > S_{k-1} > S_k = \langle 1 \rangle$ such that $S_{j-1} = S_j \leftthreetimes P_j$ where P_j is a Sylow p_j-subgroup of K and S_j is a normal Sylow p_j'-subgroup of K. We will prove this assertion using induction in $|\Pi(K)| = k$. If $k = 1$, then K is a p_1-subgroup and all is proved. Assume now that $k > 1$. The subgroup S_{k-1} is the normal Sylow p_k-subgroup of K. Then every its cyclic subgroup is subnormal in K, and by Corollary 1, every cyclic subgroup of S_{k-1} is normal in K. We have now $\Pi(K/S_{k-1}) = \{p_1, \ldots, p_{k-1}\}$, and therefore by the induction hypothesis K/S_{k-1} is supersoluble, which proves the result.

Corollary 7. *Let G be a group, K be a locally finite subgroup of G. If every cyclic subgroup of G is monopronormal in G, then K is locally supersoluble.*

Corollary 8. *Let G be a locally finite group. If every cyclic subgroup of G is monopronormal in G, then any Sylow $2'$-subgroup of G is normal.*

Proof. Let $\mathfrak{L}$ be a local family of G consisting of finite subgroups. If $L \in \mathfrak{L}$, then by Lemma 3, a Sylow $2'$-subgroup of L is normal in L. Since it is valid for each $L \in \mathfrak{L}$, then the Sylow $2'$-subgroup of G is normal in G.

Let G be a group and $\mathfrak{R}^{\mathfrak{LN}}$ be a family of all normal subgroups H of G such that G/H is locally nilpotent. Then the intersection $\bigcap \mathfrak{R}^{\mathfrak{LN}} = R^{\mathfrak{LN}}$ is called the *locally nilpotent residual* of G. It is not difficult to prove that if G is locally finite, then $G/R^{\mathfrak{LN}}$ is locally nilpotent.

Corollary 9. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then $2 \notin \Pi(L)$.*

Proof. Let D be a Sylow $2'$-subgroup of G. By Corollary 8, D is normal in G. The factor-group G/D is a locally finite 2-group, in particular, it is locally nilpotent. It follows that $L \leq D$.

Lemma 5. *Let G be a locally finite group. If every cyclic subgroup of G is monopronormal in G, then the derived subgroup $[G, G]$ is locally nilpotent.*

Proof. Indeed, by Corollary 7, G is locally supersoluble. In particular, G is locally soluble and therefore G has a chief series $\mathfrak{S}$ whose factors are abelian (see, e.g., [9, §58]). The fact that G is locally supersoluble implies that all factors of the chief series $\mathfrak{S}$ have prime orders [10, Lemma 7]. Let

$$\begin{gathered}\mathfrak{H} = \{H \in \mathfrak{S} \mid \text{there exists a subgroup } H^{\triangledown} \in \mathfrak{S} \\ \text{such that } H/H^{\triangledown} \text{ is a } G\text{-chief factor}\}.\end{gathered}$$

Since the factor $H/H^{\triangledown}$ is cyclic, $G/C_G(H/H^{\triangledown})$ is abelian for every $H \in \mathfrak{H}$.

Let $K = \bigcap_{H \in \mathfrak{H}} C_G(H/H^{\triangledown})$. By Remak's theorem we obtain an imbedding $G/K \hookrightarrow \mathbf{Cr}_{H \in \mathfrak{H}} G/C_G(H/H^{\triangledown})$, which shows that G/K is abelian. It follows that $[G, G] \leq K$. This inclusion shows that each factor $H/H^{\triangledown}$ is central in $[G, G]$ for each $H \in \mathfrak{H}$, $H \leq [G, G]$. Thus $[G, G]$ has a central series. Being locally finite, $[G, G]$ is locally nilpotent.

Corollary 10. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then L is locally nilpotent.*

Corollary 11. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then L is abelian and every subgroup of L is G-invariant.*

Proof. Indeed, by Corollary 9, L is a $2'$-subgroup. Using Corollaries 3, 4 and 10 we obtain that L is abelian. Finally, Corollary 1 proves that every subgroup of L is normal in G.

Corollary 12. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then $G/C_G(L)$ is abelian.*

Proof. By Corollary 11, every subgroup of L is G-invariant. It follows that $G/C_G(L)$ is abelian (see, for example [11, Theorem 1.5.1]).

Lemma 6. *Let G be a locally finite group and L be the locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then G/L is a Dedekind group.*

Proof. Let $p \in \Pi(G/L)$ and let P/L be a Sylow p-subgroup of G/L. Choose two arbitrary elements $xL, yL \in P/L$. Since G is locally finite, G/L is locally nilpotent, so that $\langle xL, yL\rangle = K/L$ is a finite p-subgroup. Then there exists a finite subgroup F such that $K = FL$. Choose a Sylow p-subgroup V of F. Then $V(F \cap L)/(F \cap L)$ is a Sylow p-subgroup of $F/(F \cap L)$. Since $F/(F \cap L) \cong FL/L$ is a p-group, $V(F \cap L) = F$, and $VL = K$. By Corollary 3, V is a Dedekind group. It follows that $VL/L = K/L$ is also a Dedekind group. In turn, it follows that $\langle xL\rangle^{yL} = \langle xL\rangle$. Since this is true for each element $yL \in P/L$, $\langle xL\rangle$ is normal in P/L. Hence P/L is a Dedekind group.

Lemma 7. *Let G be a locally finite group, p be an odd prime and P be a p-subgroup of G. Suppose that $N_G(P)$ contains a p'-element x such that $[P, x] \neq \langle 1\rangle$. If every cyclic subgroup of G is monopronormal in G, then every subgroup of P is $\langle x\rangle$-invariant, $P = [P, x]$, and $C_P(x) = \langle 1\rangle$.*

Proof. By Corollaries 3 and 4, P is abelian. By [12, Proposition 2.12], $P = [P, x] \times C_P(x)$. Suppose that $C_P(x) \neq \langle 1\rangle$, and choose in $C_P(x)$ an element c of order p. Let now a be an element of $[P, x]$ having order p. Lemma 2 shows that every subgroup of P is $\langle x\rangle$-invariant. Since $a \notin C_P(x)$, $a^x = a^d$, where d is a p'-number. Moreover, d is not congruent to $1(\mathrm{mod}\ p)$. We have $(ac)^x = a^x c^x = a^x c$. On the other hand, since $ac \notin C_P(x)$, $(ac)^x = (ac)^t$ where t is also p'-number such that t is not congruent with $1(\mathrm{mod}\ p)$. Hence $a^d c = (ac)^t = a^t c^t$, and therefore $d \equiv t(\mathrm{mod}\ p)$ and $t \equiv 1(\mathrm{mod}\ p)$. This contradiction proves that $C_P(x) = \langle 1\rangle$, and hence $[P, x] = P$.

Lemma 8. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then $\Pi(L) \cap \Pi(G/L) = \varnothing$.*

Proof. Suppose the contrary. Let there exists a prime p such that $p \in \Pi(L) \cap \Pi(G/L)$. The inclusion $p \in \Pi(L)$ together with Corollary 9 shows that $p \neq 2$. Let P be a Sylow p-subgroup of L and $K = N_G(P)$. Suppose that $K = G$. The subgroup L is abelian by Corollary 11, so that $L = P \times Q$ where Q is a Sylow p'-subgroup of L. By Lemma 6, G/L is a Dedekind group, in particular, it is nilpotent. In the factor-group G/Q we have $L/Q = PQ/Q \leq \zeta(G/Q)$. It follows that G/Q is nilpotent, that contradicts the choice of L. This contradiction shows that $K \neq G$. Since $p \neq 2$, Corollary 4 shows that Sylow p-subgroups of G are abelian. Then from $K \neq G$ we obtain, that K contains some p'-element x. Without loss of generality we can suppose that x is an r-element for some prime r. Since G/L is a Dedekind group and $p \in \Pi(G/L)$, G/L contains a p-element $vL \in \zeta(G/L)$. It follows that $[v, x] \in L$. Without loss of generality we can suppose that v is a p-element. Put $H = \langle x, v, P\rangle = P\langle x, v\rangle$. Then the index $|H : P|$ is finite, so that the Sylow $\{p, r\}'$-subgroup R of H is finite. The isomorphism $H/(H \cap L) \cong HL/L = \langle x, v\rangle L/L = \langle xL, vL\rangle$ implies that $\Pi(H/(H \cap L)) = \{p, r\}$, which implies that $R \leq H \cap L$. Corollary 11 shows that R is G-invariant, in particular, R is normal in H. The finiteness of R implies that $H/C_H(R)$ is finite, therefore $H = R \leftthreetimes V$, where V is a Sylow $\{p, r\}$-subgroup of H [13, Theorem 2.4.5]. Clearly, $V \cap L = P$, and $V/P = V/(V \cap L) \cong VL/L = \langle xL\rangle \times \langle vL\rangle$ is an abelian subgroup of order rp. Therefore, without loss of generality, we can suppose that $P_1 = \langle P, v\rangle$ is a normal

Sylow p-subgroup of V and $V = P_1 \leftthreetimes \langle x \rangle$. Moreover, $[V, x] \leq P$. On the other hand, the choice of x implies that $[V, x] \neq \langle 1 \rangle$. Then by Lemma 7, $[V, x] = V$, and we obtain a contradiction. This contradiction proves that P is a Sylow p-subgroup of the entire group G.

Corollary 13. *Let G be a locally finite group and L be the locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then every subgroup of $C_G(L)$ is G-invariant.*

Proof. By Lemma 8 L is the Sylow π-subgroup of G, where $\pi = \Pi(L)$. By [14, Theorem 7] $C_G(L) = L \times V$, where $V = \mathbf{O}_{\pi'}(C_G(L))$. By Corollary 11, every subgroup of L is G-invariant. Therefore, it is enough to prove that every subgroup of V is G-invariant. Let U be an arbitrary subgroup of V. Since V is G-invariant, $[U, G] \leq V$. On the other hand, by Lemma 6, G/L is a Dedekind group, so that $[U, G] \leq UL$. Thus we have $[U, G] \leq V \cap UL = U(V \cap L) = U$.

3. Proof of main result

Theorem 1. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is monopronormal in G, then the following conditions hold:*

(i) *L is abelian;*

(ii) $2 \notin \Pi(L)$ *and* $\Pi(L) \cap \Pi(G/L) = \varnothing$;

(iii) *G/L is a Dedekind group;*

(iv) *every subgroup of $C_G(L)$ is G-invariant.*

Conversely, if a group G satisfies conditions (i)-(iv), every cyclic subgroup of G is monopronormal in G.

Proof. Condition (i) follows from Corollary 11. Condition (ii) follows from Corollary 9 and Lemma 8. Condition (iii) follows from Lemma 6. Condition (iv) follows from Corollary 13.

Conversely, suppose that a group G satisfies conditions (i)-(iv). Let H be an arbitrary cyclic (and hence finite cyclic) subgroup of G. Put $C = C_G(L)$. By condition (iv) the intersection $H \cap C$ is G-invariant. In particular, if $H \leq C$, then H is normal in G. In particular, H is monopronormal in G. Suppose now that $H \neq H \cap C$. Clearly, it is enough to prove now that $H/(H \cap C)$ is monopronormal in $G/(H \cap C)$. This shows that without loss of generality we can suppose that $H \cap C = \langle 1 \rangle$. By condition (i) and (iv), every subgroup of L is G-invariant. It follows that $G/C_G(L) = G/C$ is abelian (see, for example [11, Theorem 1.5.1]). Then $H \cong H/(H \cap C) \cong HC/C$ is abelian. From condition (ii) we obtain that $\Pi(L) \cap \Pi(H) = \varnothing$.

Let x be an arbitrary element of G. Put $K = \langle H, x\rangle$ and $\pi = \Pi(H)$. Let $x \in L$. Then $K = \langle x\rangle^H \leftthreetimes H$. This means that H is a Sylow π-subgroup of K. Moreover, H is not normal in K, which implies that H is pronormal in K. This fact shows that $N_K(H)$ is abnormal in K. Thus, $N_K(H)^K = K$.

Let $x \notin L$. Put $L_1 = K \cap L$ and $\pi = \Pi(H)$. By condition (iii) L_1H is normal in K. The equation $\Pi(L) \cap \Pi(H) = \varnothing$ implies that H is a Sylow π-subgroup of L_1H. This means that H is pronormal in L_1H. Since $K = L_1HN_K(H) = L_1N_K(H)$, H is pronormal in K, and we again obtain that $N_K(H)^K = K$. Hence in any case, H is monopronormal in G.

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