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I. Ya. Subbotin*

* National University, Los Angeles, USA
E-mail: isubboti@nu.edu
ORCID ID: https://orcid.org/0000-0002-6242-3995

Methods of group theory in Leibniz algebras: some compelling results

Abstract. The theory of Leibniz algebras has been developing quite intensively. Most of the results on the structural features of Leibniz algebras were obtained for finite-dimensional algebras and many of them over fields of characteristic zero. A number of these results are analogues of the corresponding theorems from the theory of Lie algebras. The specifics of Leibniz algebras, the features that distinguish them from Lie algebras, can be seen from the description of Leibniz algebras of small dimensions. However, this description concerns algebras over fields of characteristic zero. Some reminiscences of the theory of groups are immediately striking, precisely with its period when the theory of finite groups was already quite developed, and the theory of infinite groups only arose, i.e., with the time when the formation of the general theory of groups took place. Therefore, the idea of using this experience naturally arises. It is clear that we cannot talk about some kind of similarity of results; we can talk about approaches and problems, about application of group theory philosophy. Moreover, every theory has several natural problems that arise in the process of its development, and these problems quite often have analogues in other disciplines. In the current survey, we want to focus on such issues: our goal is to observe which parts of the picture involving a general structure of Leibniz algebras have already been drawn, and which parts of this picture should be developed further.

Key words: Leibniz algebra, Lie algebra, cyclic subalgebra, left center, right center, center of a Leibniz algebra, nilpotent subalgebra, Abelian subalgebra, extraspecial subalgebra.

Анотація. Теорія алгебр Лейбніца розвивалася досить інтенсивно. Більшість результатів щодо структурних особливостей алгебр Лейбніца були отримані для скінченновимірних алгебр, і багато з них над полями нульової характеристики. Частина цих результатів є аналогами відповідних теорем з теорії алгебр Лі. Специфіку алгебр Лейбніца, особливості, що відрізняють їх від алгебр Лі, можна побачити з опису алгебр Лейбніца малих вимірностей. Однак цей опис стосується алгебр над полями нульової характеристики. Деякі спогади про теорію груп відразу кидаються в очі, а саме спогади, пов'язані з тим періодом, коли теорія скінченних груп була вже досить розвиненою, а теорія нескінченних груп лише виникла, тобто з тим часом, коли відбувалося становлення загальної теорії груп. Тому закономірно виникає ідея використання цього досвіду. Зрозуміло, що не можна говорити про якусь подібність результатів; ми можемо говорити про підходи та задачі, про застосування філософії теорії груп. Більше того, кожна теорія має низку природних проблем, які виникають у процесі її розвитку, і ці проблеми досить часто мають аналоги в інших дисциплінах. У даній оглядовій статті зосередимось на споспостереженні того, які частини малюнка із загальною структурою алгебр Лейбніца вже були намальовані, а які частини цієї картини слід розвивати далі.

Ключові слова: алгебра Лейбніца, алгебра Лі, циклічна підалгебра, лівий центр, правий центр, центр алгебри Лейбніца, нільпотентна підалгебра, абелева підалгебра, екстраспеціальна підалгебра.

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1. Introduction and some remarks

Let L be an algebra over a field F with the binary operations $+$ and $[,]$. Then L is called a *Leibniz algebra* (more precisely a *left Leibniz algebra*) if it satisfies the Leibniz identity

$$[[a, b], c] = [a, [b, c]] - [b, [a, c]]$$

for all $a, b, c \in L$.

Another form of this identity is: $[a, [b, c]] = [[a, b], c] + [b, [a, c]]$.

An algebra R over a field F is called *right Leibniz* if it satisfies the Leibniz identity

$$[a, [b, c]] = [[a, b], c] - [[a, c], b]$$

for all $a, b, c \in L$.

Note at once that the classes of left Leibniz algebras and right Leibniz algebras are different. The following simple example justifies it.

Let F be an arbitrary field and L be a vector space over F having a basis $\{a, b\}$. Define the operation $[,]$ on L by the following rule:

$$[a, a] = [a, b] = b, [b, a] = [b, b] = 0.$$

It is not hard to check that L becomes a left Leibniz algebra. But

$$0 = [[a, a], a] \neq [[a, a], a] + [a, [a, a]] = [a, b] = b.$$

Let R be a right Leibniz algebra, then put $[\![a, b]\!] = [b, a]$. Then we have

$$[\![[\![a, b]\!], c]\!] = [c, [b, a]] = [[c, b], a] - [[c, a], b] = \\ = [\![a, [\![b, c]\!]]\!] - [\![b, [\![a, c]\!]]\!].$$

Thus, this substitution leads us to a left Leibniz algebra. Similarly, we can make a transfer from a left Leibniz algebra to a right Leibniz algebra.

We prefer to work with the left Leibniz algebras. Thus, further, the term a Leibniz algebra stands for a left Leibniz algebra.

Note at once that if L is a Lie algebra, then

$$[[a, b], c] + [[b, c], a] + [[c, a], b] = 0.$$

It follows that

$$[[a, b], c] = -[[b, c], a] - [[c, a], b] = [a, [b, c]] + [b, [c, a]] = [a, [b, c]] - [b, [a, c]],$$

which shows that every Lie algebra is a Leibniz algebra.

Conversely, suppose that $[a, a] = 0$ for each element $a \in L$. Then for arbitrary elements $a, b \in L$ we have

$$0 = [a + b, a + b] = [a, a] + [a, b] + [b, a] + [b, b] = [a, b] + [b, a].$$

It follows that $[a, b] = -[b, a]$. Then we obtain

$$0 = [[a, b], c] - [a, [b, c]] + [b, [a, c]] = \\ = [[a, b], c] + [[b, c], a] - [[a, c], b] = \\ = [[a, b], c] + [[b, c], a] + [[c, a], b]$$

for all $a, b, c \in L$. Thus, Lie algebras can be characterized as Leibniz algebras in which $[a, a] = 0$ for every element $a \in L$.

Leibniz algebras first appeared in the paper of A. Blokh [3]. In this paper, A. Blokh used the term *D-algebras*. After two decades, a real interest to Leibniz algebras significantly rose. It is happened thanks to the work of J.-L. Loday [18]. J.-L. Loday "rediscovered" these algebras and used the term *Leibniz algebras* for them.

In [19] J.-L. Loday and T. Pirashvili began the systematic study of properties of Leibniz algebras.

Since that, the theory of Leibniz algebras has developed very intensively in many different directions. Some of the results of this theory were presented in the recent book of Sh.A. Ayupov, B.A. Omirov, I.S. Rakhimov [2].

The Leibniz algebras appeared to be naturally related to several areas such as differential geometry, homological algebra, classical algebraic topology, algebraic K-theory, loop spaces, noncommutative geometry, and so on. They found some applications in physics.

In the theory of Lie algebras, there is a large part, in which questions like those that arise in group theory are considered. I. Stewart called it “Infinite-dimensional Lie algebras in the spirit of the infinite group theory”. It is not just direct analogies since the final results were not always completely similar to the parallel results in the group theory. It is a more comprehensive consideration of problems, approaches, and setting tasks. Nevertheless, this part of the theory of Lie algebras is developed very intensively, there is a huge array of articles and several books. Take into account that the Lie algebras are exactly the anticommutative Leibniz algebras. If you look for some parallels, you will notice that the relationships between Leibniz algebras and Lie algebras in some ways resemble the relationships between non-Abelian and Abelian groups. Note that a very large part of articles concerned Leibniz algebras dealt with only finite-dimensional Leibniz algebras, and in most of these articles the algebras were considered over a field of characteristic 0. This situation is very similar to that which developed in the theory of groups at the beginning of the appearance of the theory of infinite groups. Therefore, it is natural to use the rich experience that group theory has. Here we are not talking about results, but about approaches and philosophies. There are similar concepts in various algebraic structures, therefore similar problems arise there. For example, in any algebraic structure, one of the first tasks is the study of substructures generated by a single element. In Lie algebras, the situation is trivial: a cyclic subalgebra of a Lie algebra L, generated by an element a, coincides with a subspace, generated by a. In contrast to Lie algebras, the situation with cyclic subalgebras in Leibniz algebras turned out to be very difficult. Description of cyclic Leibniz algebras has been obtained in the paper [5].

For the case of the field of complex numbers, the description of cyclic finite-dimensional Leibniz algebras was obtained in the paper [24]; however, it does not show the structure of cyclic Leibniz algebras.

Another natural problem that immediately arises is the study of the structure of Leibniz algebras, whose subalgebras are ideals. In group theory, a similar problem was solved a very long time ago in the classical papers of R. Dedekind and R. Baer. The Lie algebras with this property are abelian. But for Leibniz algebras, the structure of algebras, whose subalgebras are ideals, is far from being plain. The structure of such Leibniz algebras was described in the paper [9] by L.A. Kurdachenko, N.N. Semko, and I.Ya. Subbotin.

Such an algebra L has the following structure: $L = E \oplus Z$ where Z is a subalgebra of the center of L and E is an extraspecial algebra such that $[x, x] \neq 0$ for each element $x \notin \zeta(E)$.

A Leibniz algebra L is called an *extraspecial* algebra, if it satisfies the following condition:

- $\zeta(L)$ is non-trivial and has dimension 1;
- $L/\zeta(L)$ is abelian.

The *center* $\zeta(L)$ of a Leibniz algebra L is defined in the following way:

$$\zeta(L) = \{x \in L|\ [x, y] = 0 = [y, x] \text{ for each element } y \in L\}.$$

Here is the list of some important steps done on the way of developing Leibniz Algebra systematic theory which is parallel to group theory.

1. In the article [5] a description of the structure of cyclic Leibniz algebras and a description of minimal Leibniz algebras, i.e. Leibniz algebras, all proper subalgebras of which are Lie algebras, were obtained.
2. A study of Leibniz algebras, all proper subalgebras of which are ideals one can find in [9].
3. In [6] the Leibniz algebras, all proper subalgebras of which are either ideals or have a zero kernel were described.
4. The Leibniz algebras in which the relation "to be an ideal" is transitive were described in [12].
5. The articles [13] and [14] are dedicated to Leibniz algebras, all proper subalgebras of which are either ideals or contraideals.
6. The article [10] is dedicated to analysis of the influence of anticommutativity on the structure of Leibniz algebras.
7. A proof of the existence of a locally nilpotent radical in Leibniz algebras and introduction of generalized nilpotent classes of Leibniz algebras one can find in [11].
8. The article [15] is dedicated to description of Leibniz algebras, all proper subalgebras of which are either left ideals or contraideals.
9. The articles [16] and [17] developed an analogue of Schur's theorem and its generalizations.

2. Nilpotency of Leibniz algebras

To give the reader a flavor of the listed above results, in this section we consider some results related to the concept of nilpotency. This concept arises both in the theory of groups and in the theory of rings and algebras (associative and non-associative). In the theory of Leibniz algebras this concept arises as follows.

Every Leibniz algebra L has the following specific ideal. Denote by $\mathrm{Leib}(L)$ the subspace generated by the elements $[a, a]$, $a \in L$. We note that $\mathrm{Leib}(L)$ is an ideal of L. Indeed, for arbitrary elements $a, x \in L$ we have

$$[a, [a, x]] = [[a, a], x] + [a, [a, x]], \text{ so } [[a, a], x] = 0.$$

Furthermore,

$$[x+[a,a],x+[a,a]] = [x,x]+[x,[a,a]]+[[a,a],x]+[[a,a],[a,a]] = \\ = [x,x]+[x,[a,a]].$$

It follows that $[x,[a,a]] = [x+[a,a],x+[a,a]]-[x,x] \in \mathrm{Leib}(L)$.

Put $K = \mathrm{Leib}(L)$. Then in the factor-algebra L/K we have

$$[a+K,a+K] = [a,a]+K = K$$

for each element $a \in L$. By above, we obtain that L/K is a Lie algebra. Conversely, suppose that H is an ideal of L such that L/H is a Lie algebra. Then

$$H = [a+H,a+H] = [a,a]+H,$$

which implies that $[a,a] \in H$ for every element $a \in L$. Then $\mathrm{Leib}(L) \leqslant H$.

The ideal $\mathrm{Leib}(L)$ is called the Leibniz kernel of algebra L.

We note the following important property of the Leibniz kernel:

$$[[a,a],x] = [a,[a,x]]-[a,[a,x]] = 0.$$

This property shows that $\mathrm{Leib}(L)$ is an abelian subalgebra of L.

Let L be a Leibniz algebra. Define the *lower central series* of L

$$L = \gamma_1(L) \geqslant \gamma_2(L) \geqslant \dots \gamma_\alpha(L) \geqslant \gamma_{\alpha+1}(L) \geqslant \dots \gamma_\delta(L)$$

by the following rule: $\gamma_1(L) = L$, $\gamma_2(L) = [L,L]$, and, recursively, $\gamma_{\alpha+1}(L) = [L,\gamma_\alpha(L)]$ for all ordinals α and $\gamma_\lambda(L) = \bigcap_{\mu<\lambda} \gamma_\mu(L)$ for the limit ordinals λ. The last term $\gamma_\delta(L)$ is called the *lower hypocenter* of L. We have $\gamma_\delta(L) = [L,\gamma_\delta(L)]$.

Since $\zeta(L)$ is an ideal of L, we can consider the factor-algebra $L/\zeta(L)$.

A Leibniz algebra L is called *nilpotent* if there exists a positive integer k such that $\gamma_k(L) = \langle 0\rangle$. More precisely, L is said to be *nilpotent of nilpotency class c* if $\gamma_{c+1}(L) = \langle 0\rangle$, but $\gamma_c(L) \neq \langle 0\rangle$. We denote by $\mathrm{ncl}(L)$ the nilpotency class of L.

In some algebraic structures another definition of nilpotency based on the concept of the (upper) central series is used. In fact, suppose that L is a nilpotent Leibniz algebra and $\gamma_{k+1}(L) = \langle 0\rangle$. For each factor $\gamma_j(L)/\gamma_{j+1}(L)$ we have $[L,\gamma_j(L)] = \gamma_{j+1}(L)$ and $[\gamma_j(L),L] \leqslant \gamma_{j+1}(L)$, and this leads us to the following concepts.

Let A, B be ideals of L such that $A \leqslant B$. The factor B/A is called *central* (in L) if $[L,B],[B,L] \leqslant A$.

Starting from the center we can define the *upper central series*

$$\langle 0\rangle = \zeta_0(L) \leqslant \zeta_1(L) \leqslant \dots \zeta_\alpha(L) \leqslant \zeta_{\alpha+1}(L) \leqslant \dots \zeta_\gamma(L) = \zeta_\infty(L)$$

of the Leibniz algebra L by the following rule: $\zeta_1(L) = \zeta(L)$ is the center of L, and recursively $\zeta_{\alpha+1}(L)/\zeta_\alpha(L) = \zeta(L/\zeta_\alpha(L))$ for all ordinals α, and

$\zeta_\lambda(L) = \bigcup_{\mu<\lambda} \zeta_\mu(L)$ for limit ordinals λ. By definition, each term of this series is an ideal of L. The last term $\zeta_\infty(L)$ of this series is called the *upper hypercenter* of L. A Leibniz algebra L is said to be *hypercentral* if it coincides with the upper hypercenter. Denote by $\mathrm{zl}(L)$ the length of upper central series of L.

It is a well-known that in nilpotent Lie algebras and nilpotent groups the lower and the upper central series have the same length.

Let

$$\langle 0 \rangle = C_0 \leqslant C_1 \leqslant \dots C_\alpha \leqslant C_{\alpha+1} \leqslant \dots C_\gamma = L$$

be an ascending series of ideals of a Leibniz algebra L. This series is called *central* if $C_{\alpha+1}/C_\alpha \leqslant \zeta(L/C_\alpha)$ for each ordinal $\alpha < \gamma$. In other words, $[C_{\alpha+1}, L], [L, C_{\alpha+1}] \leqslant C_\alpha$ for each ordinal $\alpha < \gamma$.

We note the following properties of central series [16].

Theorem 1. *Let L be a Leibniz algebra over a field F, and*

$$\langle 0 \rangle = C_0 \leqslant C_1 \leqslant \dots \leqslant C_n = L$$

be a finite central series of L. Then

(i) *$\gamma_j(L) \leqslant C_{n-j+1}$, so that $\gamma_{n+1}(L) = \langle 0 \rangle$.*

(ii) *$C_j \leqslant \zeta_j(L)$, so that $\zeta_n(L) = L$.*

(iii) *If j, k are positive integer such that $k \geqslant j$, then*

$$[\gamma_j(L), \zeta_k(L)], [\zeta_k(L), \gamma_j(L)] \leqslant \zeta_{k-j}(L).$$

As a corollary we obtain

Corollary 1. *Let L be a Leibniz algebra over a field F and suppose that L has a finite central series*

$$\langle 0 \rangle = C_0 \leqslant C_1 \leqslant \dots \leqslant C_n = L.$$

Then L is nilpotent and $\mathrm{ncl}(L) \leqslant n$. Furthermore, the upper central series of L is finite, $\zeta_\infty(L) = L$, $\mathrm{zl}(L) \leqslant n$. Moreover, $\mathrm{ncl}(L) = \mathrm{zl}(L)$.

The last result shows that a Leibniz algebra L is nilpotent if and only if there is a positive integer k such that $L = \zeta_k(L)$. The least positive integer having this property coincides with nilpotency class of L. So, as in the cases of Lie algebras and groups, the definition of nilpotency can be given here using the notion of the upper central series.

It will be appropriate to note that the Leibniz algebra L can be associative.

Theorem 2. *Let L be a Leibniz algebra over a field F. Then L is associative if and only if $[L, L] \leqslant \zeta(L)$.*

The concepts of upper and lower central series immediately leads us to the mentioned classes of Leibniz algebras.

A Leibniz algebra L is said to be *hypercentral* if it coincides with the upper hypercenter.

A Leibniz algebra L is said to be *hypocentral* if its lower hypocenter is trivial.

In the case of finite dimensional algebras, these two concepts coincide, but, in general, these two classes are very different.

Thus, for finitely generated hypercentral Leibniz algebras we have (see [11])

Theorem 3. *Let L be a finitely generated Leibniz algebra over a field F. If L is hypercentral, then L is nilpotent. Moreover, L has finite dimension. In particular, a finitely generated nilpotent Leibniz algebra has finite dimension.*

This result is an analog of a similar group theoretical result proved by A.I. Mal'cev [20].

At the same time, a finitely generated hypocentral Leibniz algebra can have infinite dimension. A simple example, which shows it is a cyclic Leibniz algebra, generated by an element of infinite depth.

A Leibniz algebra L is said to be *locally nilpotent* if every finite subset of L generates a nilpotent subalgebra.

That is why, hypercentral Leibniz algebras give us examples of locally nilpotent algebras.

We obtained the following characterization of hypercentral Leibniz algebras.

Theorem 4. *Let L be a Leibniz algebra over a field F. Then L is hypercentral if and only if for each element $a \in L$ and every countable subset $\{x_n |\ n \in \mathbb{N}\}$ of elements of L there exists a positive integer k such that all commutators $[x_1, \ldots, x_j, a, x_{j+1}, \ldots, x_k]$ are zeros for all j, $0 \leqslant j \leqslant k$.*

As a corollary we obtain

Corollary 2. *Let L be a Leibniz algebra over a field F. Then L is hypercentral if and only if every subalgebra of L having finite or countable dimension is hypercentral.*

These results are analogs to some group theoretical results of S.N. Chernikov.

Let L be a Leibniz algebra. If A, B are nilpotent ideals of L, then their sum $A + B$ is a nilpotent ideal of L. This result has been proved in the paper [4].

In this connection, the following question arises: *if a similar assertion is valid for locally nilpotent ideals?* For Lie algebras this assertion takes place. It was shown by B. Hartley in the paper [7].

We have an affirmative answer to this question.

Theorem 5. *Let L be a Leibniz algebra over a field F, A, B be locally nilpotent ideals of L. Then $A + B$ is locally nilpotent.*

As corollaries we obtain

Corollary 3. *Let L be a Leibniz algebra over a field F and $\mathfrak{S}$ be a family of locally nilpotent ideals of L. Then a subalgebra generated by $\mathfrak{S}$ is locally nilpotent.*

Corollary 4. *Let L be a Leibniz algebra over a field F. Then L has the greatest locally nilpotent ideal.*

Let L be a Leibniz algebra over field F. The greatest locally nilpotent ideal of L is called the *locally nilpotent radical* of L and will be denoted by $\mathrm{Ln}(L)$.

These results are analogues of the results in groups proven by K.A. Hirsch [8] and B.I. Plotkin [22, 23].

The subalgebra $\mathrm{Nil}(L)$ generated by all nilpotent ideals of L is called the nil-radical of L. If $L = \mathrm{Nil}(L)$, then L is called a *Leibniz nil-algebra.* Every nilpotent Leibniz algebra is a nil-algebra, but converse is not true even for a Lie algebra. Every Leibniz nil-algebra is locally nilpotent, but converse is not true even for a Lie algebra. Moreover, there exists a Lie nil-algebra, which is not hypercentral. There is a corresponding example in Chapter 6 of the book [1].

Note the following important properties of locally nilpotent Leibniz algebras.

Theorem 6. *Let L be a locally nilpotent Leibniz algebra over a field F.*

(i) *If A, B, $A \leqslant B$ are the ideals of L such that B/A is L-chief, then B/A is central in L (that is $B/A \leqslant \zeta(L/A)$). In particular,* $\dim_F(B/A) = 1$.

(ii) *If A is a maximal subalgebra of L, then A is an ideal of L.*

Let L be a Leibniz algebra over the field F and H a subalgebra of L. The *idealizer* of H is defined by the following rule:

$$\mathrm{I}_L(H) = \{x \in L |\ [h, x], [x, h] \in H \text{ for all } h \in H\}.$$

It is possible to prove that the idealizer of H is a subalgebra of L. If L is a hypercentral (in particular, nilpotent) Leibniz algebra, then $H \neq \mathrm{I}_L(H)$. This leads us to the following class of Leibniz algebras.

Let L be a Leibniz algebra over field F. We say that L satisfies the *idealizer condition* if $\mathrm{I}_L(A) \neq A$ for every proper subalgebra A of L.

A subalgebra A is called *ascendant* in L, if there is an ascending chain of subalgebras

$$A = A_0 \leqslant A_1 \leqslant \dots A_\alpha \leqslant A_{\alpha+1} \leqslant \dots A_\gamma = L$$

such that A_α is an ideal of $A_{\alpha+1}$ for all $\alpha < \gamma$.

It is possible to prove that L satisfies the idealizer condition if and only if every subalgebra of L is ascendant.

The last result is the following

Theorem 7. *Let L be a Leibniz algebra over a field F. If L satisfies the idealizer condition then L is locally nilpotent.*

This result is an analog to famous result proved by B.I. Plotkin for groups [21].

Again, it should be noted that the Leibniz algebras with the idealizer condition will form a proper subclass of the class of locally nilpotent Leibniz algebras. It happens since this is already the case for Lie algebras. A corresponding example could be found in Chapter 6 of the book [1].

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