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On an analogue of Schur's theorem for Leibniz n-algebras

Abstract. In this paper, we investigate relationships between certain important subalgebras of Leibniz n-algebras. In particular, we establish a close connection between the central factor-algebra of a Leibniz n-algebra and its derived ideal. As an application, we prove an analogue of the classical group-theoretic Schur's theorem for Leibniz n-algebras. The obtained results continue a long line of research on Schur-type theorems in various algebraic structures and generalize known related results from the theories of Leibniz algebras and Lie algebras.

Key words: center, derived ideal, Leibniz n-algebra, Lie n-algebra

Анотація. У цій статті досліджуються зв'язки між деякими важливими підалгебрами n-алгебр Лейбніца. Зокрема, встановлено тісний зв'язок між центральною фактор-алгеброю n-алгебри Лейбніца та її похідним ідеалом. На цій основі для n-алгебр Лейбніца доведено аналог класичної теоретико-групової теореми Шура. Отримані результати продовжують низку досліджень аналогів теореми Шура в різних алгебричних структурах, що активно проводяться алгебраїстами протягом останніх десятиліть, та узагальнюють відомі твердження з теорій алгебр Лейбніца й алгебр Лі.

Ключові слова: центр, похідний ідеал, n-алгебра Лейбніца, n-алгебра Лі

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1. Introduction.

One of the classical results in group theory is Schur's theorem, proved by B.H. Neumann in 1951 [19]. A detailed account of the history of this remarkable result can be found in [15]. This theorem states that if the central factor-group $G/\zeta(G)$ of a group G is finite, then its derived subgroup $[G, G]$ is also finite. Equivalently, if $G/\zeta(G)$ is finite, then G possesses a finite normal subgroup $H = [G, G]$ such that the factor-group G/H is abelian. Moreover, there exists a function w such that

$$|[G, G]| \leqslant w(t),$$

where $t = |G/\zeta(G)|$.

Schur's theorem has been generalized and modified in numerous directions within group theory (see, for example, [4, 5, 7, 9, 10, 20]). Furthermore, analogous results have been obtained for various algebraic structures of a different nature. In particular, analogues of Schur's theorem were established for modules [16], linear groups [3], topological groups [25], n-groups [6], associative algebras [21], Lie algebras [11, 22], Lie n-algebras [23], Lie rings [12], Leibniz algebras [8], and Poisson algebras [13]. Related results for certain 3-algebras can be found in [17, 18].

Let L be an algebra over a field F with the binary operations $+$ and $[-,-]$. Then L is called a (left) *Leibniz algebra* if it satisfies the (left) *Leibniz identity*:

$$[a,[b,c]] = [[a,b],c] + [b,[a,c]]$$

for all $a, b, c \in L$.

Ten years ago L.A. Kurdachenko, J. Otal, and O.O. Pypka proved an analogue of Schur's theorem for Leibniz algebras [8]. More precisely, it was shown that if the center $\zeta(L)$ of a Leibniz algebra L has finite codimension d, then the derived ideal $[L, L]$ is finite-dimensional and

$$\dim_F[L, L] \leqslant d^2.$$

Moreover, the authors considered a more general situation, namely the case where the codimensions of the left and right centers of a Leibniz algebra are finite. It was proved that if $\operatorname{codim}_F \zeta^{\text{left}}(L) = d$ and $\operatorname{codim}_F \zeta^{\text{right}}(L) = r$, then

$$\dim_F[L, L] \leqslant d(d + r).$$

The development of this line of research indicates that analogues of Schur's theorem hold for a wide range of algebraic structures, as evidenced by the steadily growing list of such structures. At the same time, it is well known that Schur-type results do not hold universally for all classes of groups, algebras, or related objects, which makes further investigations in this direction both natural and promising.

One of the most natural generalizations of Leibniz algebras is given by Leibniz n-algebras, which arise by replacing the binary operation with an n-ary operation satisfying appropriate identities. It is therefore natural to attempt to extend known results from the theory of Leibniz algebras to the setting of Leibniz n-algebras. However, experience shows that such extensions are not always straightforward, and additional difficulties often arise when passing from binary to n-ary structures.

The aim of the present paper is to establish an analogue of Schur's theorem for Leibniz n-algebras.

2. Preliminary results and remarks.

The theory of Leibniz algebras has been one of the most actively developing areas of modern algebra over the last few decades. In recent years, two monographs [1, 14] have been published, which present contemporary results of the theory and highlight the main current trends in the study of Leibniz algebras. On the other hand, the theory of Leibniz n-algebras remains much less explored, with many natural questions arising in the transition from binary Leibniz algebras to n-ary structures.

We begin with some fundamental notions.

Let L be an n-algebra over a field F, equipped with a binary operation $+$ and an n-linear bracket $[-,\dots,-]$. The algebra L is called a (left) *Leibniz n-algebra* [2] if it satisfies the following identity:

$$[b_1,\dots,b_{n-1},[a_1,\dots,a_n]] =$$
$$\sum_{i=1}^{n} [a_1,\dots,a_{i-1},[b_1,\dots,b_{n-1},a_i],a_{i+1},\dots,a_n].$$

We note that for $n = 2$, this concept coincides with the usual (left) Leibniz algebra.

If $A_1,\dots,A_n$ are subspaces of a Leibniz n-algebra L, then $[A_1,\dots,A_n]$ will denote a subspace generated by all elements $[a_1,\dots,a_n]$ where $a_i \in A_i$, $1 \leqslant i \leqslant n$. As usual, a subspace A of L is called a *subalgebra* of L if $[a_1,\dots,a_n] \in A$ for all elements $a_1,\dots,a_n \in A$.

A subalgebra A of a Leibniz n-algebra L is called the *i-th ideal* of L if

$$[a_1,\dots,a_n] \in A$$

whenever $a_{n-i+1} \in A$ and $a_j \in L$ for all $j \neq n-i+1$.

Note that in the case of ordinary Leibniz algebras, the notion of the 1st ideal coincides with that of a left ideal, while the n-th ideal coincides with the usual notion of a right ideal.

A subalgebra A of L is called an *ideal* of L (or an *n-sided ideal*) if it is an i-th ideal of L for all $1 \leqslant i \leqslant n$. If A is an ideal of L, we can consider the factor-algebra L/A. It is easy to see that L/A is also a Leibniz n-algebra.

Let L be a Leibniz n-algebra over a field F, let M be a non-empty subset of L, and let H be a subalgebra of L. Put

$$Ann^i_H(M) = \{a \in H \mid [M,\dots,a,\dots,M] = \langle 0\rangle \text{ with } a \text{ in the } i\text{-th position}\}.$$

The set $Ann^i_H(M)$ is called the *i-th annihilator* of M in H. The intersection

$$Ann_H(M) = \bigcap_{i=1}^{n} Ann^i_H(M)$$

is called the *annihilator* of M in H.

Proposition 1. Let L be a Leibniz n-algebra over a field F, let M be an ideal of L, and let H be a subalgebra of L. Then, for every $1 \leqslant k \leqslant n$, the set

$Ann_H^k(M)$ is a subalgebra of L. Consequently,

$$Ann_H(M) = \bigcap_{k=1}^{n} Ann_H^k(M)$$

is also a subalgebra of L.

Proof. Fix $k \in \{1, \ldots, n\}$ and put $A = Ann_H^k(M)$. Let $a_1, \ldots, a_n \in A$. We prove that $[a_1, \ldots, a_n] \in A$.

Take arbitrary elements $x_j \in M$ for all $j \neq k$ and consider

$$[x_1, \ldots, x_{k-1}, a_n, x_{k+1}, \ldots, x_n] = 0,$$

because $a_n \in A$.

Applying the Leibniz n-identity to the bracket

$$[a_1, \ldots, a_{n-1}, [x_1, \ldots, x_{k-1}, a_n, x_{k+1}, \ldots, x_n]],$$

we obtain

$$\begin{aligned} 0 &= [a_1, \ldots, a_{n-1}, [x_1, \ldots, x_{k-1}, a_n, x_{k+1}, \ldots, x_n]] \\ &= \sum_{\substack{j=1 \\ j \neq k}}^{n} [x_1, \ldots, x_{j-1}, [a_1, \ldots, a_{n-1}, x_j], x_{j+1}, \ldots, x_{k-1}, a_n, x_{k+1}, \ldots, x_n] \\ &\quad + [x_1, \ldots, x_{k-1}, [a_1, \ldots, a_{n-1}, a_n], x_{k+1}, \ldots, x_n]. \end{aligned}$$

Since M is an ideal of L, we have $[a_1, \ldots, a_{n-1}, x_j] \in M$ for every $j \neq k$. Hence each summand in the sum is equal to 0, because it has $a_n \in A$ in the k-th position and all remaining arguments in M. Therefore,

$$[x_1, \ldots, x_{k-1}, [a_1, \ldots, a_n], x_{k+1}, \ldots, x_n] = 0,$$

which shows that $[a_1, \ldots, a_n] \in A$. Thus $Ann_H^k(M)$ is a subalgebra of L. The last statement follows by taking intersections.

The i-th annihilators lead us to the following subspaces. Put

$$\zeta^i(L) = \{a \in L \mid [x_1, \ldots, x_{i-1}, a, x_{i+1}, \ldots, x_n] = 0 \text{ for all } x_j \in L,\ j \neq i\}.$$

The subset $\zeta^i(L)$ is called the *i-th center* of L.

Put

$$\zeta(L) = \bigcap_{i=1}^{n} \zeta^i(L).$$

The subset $\zeta(L)$ is called the *center* of L. Note that

$$\zeta^i(L) = Ann_L^i(L)$$

for each i, and therefore

$$\zeta(L) = \bigcap_{i=1}^{n} Ann_L^i(L) = Ann_L(L).$$

In general, the sets $\zeta^i(L)$ may be different and may even have different dimensions, as can already be seen in the case of ordinary (binary) Leibniz algebras (see Example 2.1 in [8]). In general, the sets $\zeta^i(L)$ need not be ideals of L. However, we have the following result.

For convenience, we set

$$\zeta^{\text{left}}(L) = \bigcap_{i=1}^{n-1} \zeta^i(L) \quad \text{and} \quad \zeta^{\text{right}}(L) = \zeta^n(L).$$

The sets $\zeta^{\text{left}}(L)$ and $\zeta^{\text{right}}(L)$ are natural analogues of the left and right centers defined for ordinary (binary) Leibniz algebras.

Proposition 2. Let L be a Leibniz n-algebra over a field F. Then the following assertions hold.

(i) For every $1 \leqslant i \leqslant n$, the set $\zeta^i(L)$ is a subalgebra of L. Consequently, the center $\zeta(L) = \bigcap_{i=1}^{n} \zeta^i(L)$ is also a subalgebra of L.

(ii) The left center $\zeta^{\text{left}}(L) = \bigcap_{i=1}^{n-1} \zeta^i(L)$ is an ideal of L.

(iii) The center $\zeta(L)$ is an ideal of L.

Proof. (i) Recall that for every $1 \leqslant i \leqslant n$ we have

$$\zeta^i(L) = Ann_L^i(L).$$

Since L is an ideal of itself and L is a subalgebra of L, Proposition 1 (applied with $M = L$ and $H = L$) implies that each $\zeta^i(L)$ is a subalgebra of L. As $\zeta(L)$ is an intersection of subalgebras, it is also a subalgebra of L.

(ii) Put

$$D = \zeta^{\text{left}}(L) = \bigcap_{i=1}^{n-1} \zeta^i(L).$$

We prove that D is an ideal of L.

Let

$$u = [x_1, \dots, x_{n-1}, v], \qquad v \in D,\ x_1, \dots, x_{n-1} \in L.$$

Fix $s \in \{1, \dots, n-1\}$ and take arbitrary $b_1, \dots, b_n \in L$. Since $v \in \zeta^s(L)$, we have

$$[b_1, \dots, b_{s-1}, v, b_{s+1}, \dots, b_n] = 0,$$

and hence

$$0 = [x_1, \dots, x_{n-1}, [b_1, \dots, b_{s-1}, v, b_{s+1}, \dots, b_n]].$$

Applying the Leibniz n-identity to this bracket, we obtain

$$\begin{aligned}0 = &\sum_{\substack{i=1\\ i\neq s}}^{n}[b_1,\dots,b_{i-1},[x_1,\dots,x_{n-1},b_i],b_{i+1},\dots,b_{s-1},v,b_{s+1},\dots,b_n]\\ &+[b_1,\dots,b_{s-1},[x_1,\dots,x_{n-1},v],b_{s+1},\dots,b_n].\end{aligned}$$

All summands in the first sum are equal to 0, since v occurs in the s-th position and $v \in \zeta^s(L)$. Therefore, $[b_1,\dots,b_{s-1},u,b_{s+1},\dots,b_n] = 0$. As $s \in \{1,\dots,n-1\}$ is arbitrary, it follows that $u \in D$, and hence $[L,\dots,L,D] \subseteq D$.

The remaining inclusions $[L,\dots,L,D,L,\dots,L] \subseteq D$ for an arbitrary position of D are proved in an analogous way.

(iii) In the same way, one can show that the center $\zeta(L)$ of L is an ideal of L, since its elements annihilate the bracket in all positions.

Let L be a Leibniz n-algebra over a field F. A linear transformation f of L is called a *derivation* of L if

$$f([a_1,\dots,a_n]) = \sum_{i=1}^{n}[a_1,\dots,a_{i-1},f(a_i),a_{i+1},\dots,a_n]$$

for all $a_1,\dots,a_n \in L$.

For arbitrary elements $a_1,\dots,a_{n-1} \in L$, consider the mapping

$$l_{a_1,\dots,a_{n-1}} : L \to L, \qquad l_{a_1,\dots,a_{n-1}}(x) = [a_1,\dots,a_{n-1},x].$$

We note some basic properties of this mapping. Let $x,y \in L$ and $\lambda \in F$. Then

$$\begin{aligned}l_{a_1,\dots,a_{n-1}}(x+y) &= [a_1,\dots,a_{n-1},x+y]\\ &= [a_1,\dots,a_{n-1},x]+[a_1,\dots,a_{n-1},y]\\ &= l_{a_1,\dots,a_{n-1}}(x)+l_{a_1,\dots,a_{n-1}}(y),\\ l_{a_1,\dots,a_{n-1}}(\lambda x) &= [a_1,\dots,a_{n-1},\lambda x] = \lambda[a_1,\dots,a_{n-1},x] = \lambda l_{a_1,\dots,a_{n-1}}(x).\end{aligned}$$

Hence, $l_{a_1,\dots,a_{n-1}}$ is a linear transformation of L.

Furthermore, for arbitrary elements $x_1,\dots,x_n \in L$, by the Leibniz identity we have

$$\begin{aligned}l_{a_1,\dots,a_{n-1}}([x_1,\dots,x_n]) &= [a_1,\dots,a_{n-1},[x_1,\dots,x_n]]\\ &= \sum_{i=1}^{n}[x_1,\dots,x_{i-1},[a_1,\dots,a_{n-1},x_i],x_{i+1},\dots,x_n]\\ &= \sum_{i=1}^{n}[x_1,\dots,x_{i-1},l_{a_1,\dots,a_{n-1}}(x_i),x_{i+1},\dots,x_n].\end{aligned}$$

These equalities show that $l_{a_1,\dots,a_{n-1}}$ is a derivation of L.

3. Main result.

As usual, let $[L^{(n)}]$ denote the derived ideal of the Leibniz n-algebra L. We now present the main result of this paper.

Theorem 1. *Let L be a Leibniz n-algebra over a field F. Assume that*

$$\mathrm{codim}_F \zeta^{\mathrm{left}}(L) = d \quad \textit{and} \quad \mathrm{codim}_F \zeta^{n}(L) = r$$

are finite. Then

$$\dim_F [L^{(n)}] \leqslant d^{n-1}(d+r).$$

Proof. Put $Z = \zeta^{\mathrm{left}}(L)$. Since $\mathrm{codim}_F(Z) = d$, we have a vector space decomposition

$$L = Z \oplus E,$$

where $\dim_F E = d$. Choose a basis $\{e_1, \ldots, e_d\}$ of E.

Let $x_1, \ldots, x_n \in L$ be arbitrary. Each element can be written in the form

$$x_k = \sum_{i=1}^{d} \alpha_{ki} e_i + z_k, \qquad \alpha_{ki} \in F,\ z_k \in Z.$$

Using multilinearity of the bracket and the definition of Z, we obtain

$$\begin{aligned}[x_1, \ldots, x_n] = & \sum_{1 \leqslant i_1, \ldots, i_n \leqslant d} \alpha_{1i_1} \cdots \alpha_{ni_n} [e_{i_1}, \ldots, e_{i_n}] \\ & + \sum_{1 \leqslant i_1, \ldots, i_{n-1} \leqslant d} \beta_{i_1 \ldots i_{n-1}} [e_{i_1}, \ldots, e_{i_{n-1}}, z],\end{aligned}$$

where $z \in Z$. Hence, $[L^{(n)}]$ is contained in the subspace generated by

$$[e_{i_1}, \ldots, e_{i_n}] \quad \text{and} \quad [e_{i_1}, \ldots, e_{i_{n-1}}, Z].$$

Fix arbitrary elements $a_1, \ldots, a_{n-1} \in L$. Consider the mapping

$$l_{a_1,\ldots,a_{n-1}} : Z \to Z, \qquad l_{a_1,\ldots,a_{n-1}}(z) = [a_1, \ldots, a_{n-1}, z].$$

As shown above, this mapping is a derivation of L and hence a linear transformation of Z. Its image is $[a_1, \ldots, a_{n-1}, Z]$ and

$$Ker(l_{a_1,\ldots,a_{n-1}}) = Ann_Z^n(a_1, \ldots, a_{n-1}),$$

where $(a_1, \ldots, a_{n-1})$ is an ordered tuple of elements $a_1, \ldots, a_{n-1}$. Since $\zeta^n(L) \subseteq Ann_L^n(a_1, \ldots, a_{n-1})$, we obtain

$$\dim_F [a_1, \ldots, a_{n-1}, Z] = \dim_F \big(Z / Ker(l_{a_1,\ldots,a_{n-1}})\big) \leqslant r.$$

In particular,

$$\dim_F [e_{i_1}, \ldots, e_{i_{n-1}}, Z] \leqslant r \quad \text{for all } 1 \leqslant i_1, \ldots, i_{n-1} \leqslant d.$$

Therefore,

$$\dim_F [L^{(n)}] \leqslant d^n + d^{n-1} r = d^{n-1}(d+r),$$

as required.

Corollary 1. *Let L be a Leibniz n-algebra over a field F. If codimension* $\operatorname{codim}_F(\zeta(L)) = d$ *is finite, then*

$$\dim_F[L^{(n)}] \leqslant d^n.$$

Proof. Since

$$\zeta(L) = \zeta^{\text{left}}(L) \cap \zeta^n(L),$$

both $\operatorname{codim}_F \zeta^{\text{left}}(L)$ and $\operatorname{codim}_F \zeta^n(L)$ are finite. Applying Theorem 1, we obtain the required estimate.

Corollary 2. *[8] Let L be a Leibniz algebra over a field F. If codimensions* $\operatorname{codim}_F \zeta^{\text{left}}(L) = d$ *and* $\operatorname{codim}_F \zeta^{\text{right}}(L) = r$ *are finite, then* $\dim_F[L, L] \leqslant d(d + r)$.

Corollary 3. *[8] Let L be a Leibniz algebra over a field F. If* $\operatorname{codim}_F \zeta(L) = d$ *is finite, then* $dim_F[L, L] \leqslant d^2$.

Recall that an n-algebra L over a field F with the binary operations $+$ and an n-linear bracket $[-, \dots, -]$ is called a *Lie n-algebra* if it satisfies the following conditions:

- Lie n-bracket is antisymmetric, that is

$$[a_1, \dots, a_n] = \operatorname{sign}(\sigma)[a_{\sigma(1)}, \dots, a_{\sigma(n)}],$$

- Lie n-bracket satisfies the generalized Jacobi identity, that is

$$[[a_1, \dots, a_n], b_1, \dots, b_{n-1}] =$$
$$\sum_{i=1}^{n} [a_1, \dots, a_{i-1}, [a_i, b_1, \dots, b_{n-1}], a_{i+1}, \dots, a_n]$$

for any $a_1, \dots, a_n, b_1, \dots, b_{n-1} \in L$ and any permutation $\sigma \in S_n$.

Corollary 4. *Let L be a Lie n-algebra over a field F. If* $\operatorname{codim}_F \zeta(L) = d$ *is finite, then*

$$\dim_F[L^{(n)}] \leqslant \binom{d}{n}.$$

Corollary 5. *[24] If L is a Lie algebra over a field F and $L/\zeta(L)$ has finite dimension t, then $[L, L]$ also has finite dimension and* $\dim_F[L, L] \leqslant \frac{t(t-1)}{2}$.

References

1. *Ayupov Sh., Omirov B., Rakhimov I.*: Leibniz Algebras: Structure and Classification. CRC Press, Taylor & Francis Group, 2020. `doi:10.1201/9780429344336`
2. *Casas J.M., Loday J.-L., Pirashvili T.*: Leibniz n-algebras. Forum Math. 2002; 14: pp. 189–207. `doi:10.1515/form.2002.009`

3. *Dixon M.R., Kurdachenko L.A., Otal J.*: Linear analogues of theorems of Schur, Baer and Hall. Int. J. Group Theory 2013; 2(1): pp. 79–89. `doi:10.22108/IJGT.2013.2441`
4. *Dixon M.R., Kurdachenko L.A., Pypka A.A.*: On some variant of theorems of Schur and Baer. Milan. J. Math. 2014; 82(2): pp. 233–241. `doi:10.1007/s00032-014-0215-9`
5. *Dixon M.R., Kurdachenko L.A., Pypka A.A.*: The theorems of Schur and Baer: a survey. Int. J. Group Theory 2015; 4(1): pp. 21–32. `doi:10.22108/IJGT.2015.7376`
6. *Galmak A.M.*: On the Schur theorem for n-ary groups. Ukr. Math. J. 2006; 58(6): pp. 730–741. `doi:10.1007/s11253-006-0106-5`
7. *Kurdachenko L.A., Otal J., Pypka A.A.*: On the structure and some numerical properties of subgroups and factor-groups defined by automorphism groups. J. Algebra Appl. 2015; 14(5): 1550075. `doi:10.1142/S0219498815500759`
8. *Kurdachenko L.A., Otal J., Pypka A.A.*: Relationships between the factors of the canonical central series of Leibniz algebras. Eur. J. Math. 2016; 2(2): pp. 565–577. `doi:10.1007/s40879-016-0093-5`
9. *Kurdachenko L.A., Otal J., Pypka A.A.*: Relationships between the factors of the upper and the lower central series of a group. B. Malays. Math. Sci. So. 2016; 39(3): pp. 1115–1124. `doi:10.1007/s40840-015-0222-1`
10. *Kurdachenko L.A., Pypka O.O.*: On some generalizations of Baer's theorem. Carpathian Math. Publ. 2014; 6(2): pp. 310–316. `doi:10.15330/cmp.6.2.310-316`
11. *Kurdachenko L.A., Pypka A.A., Subbotin I.Ya.*: On some relations between the factors of the upper and lower central series in Lie algebras. Serdica Math. J. 2015; 41(2-3): pp. 293–306.
12. *Kurdachenko L.A., Pypka A.A., Subbotin I.Ya.*: On new analogs of some classical group theoretical results in Lie rings. Infinite Group Theory, 2018; pp. 197-213. `doi:10.1142/9789813204058_0011`
13. *Kurdachenko L.A., Pypka A.A., Subbotin I.Ya.*: On extension of classical Baer results to Poisson algebras. Algebra Discrete Math. 2021; 31(1): pp. 84–108. `doi:10.12958/adm1758`
14. *Kurdachenko L.A., Pypka O.O., Subbotin, I.Ya.*: General Theory of Leibniz Algebras. Switzerland: Springer, 2024. `doi:10.1007/978-3-031-58148-9`
15. *Kurdachenko L.A., Subbotin I.Ya.*: A brief history of an important classical theorem. Adv. Group Theory Appl. 2016; 2: pp. 121–124. `doi:10.4399/97888548970148`
16. *Kurdachenko L.A., Subbotin I.Ya., Chupordia V.A.*: On the relations between the central factor-module and the derived submodule in modules over group rings. Comment. Math. Univ. Carolin. 2015; 56(4): pp. 433–445. `doi:10.14712/1213-7243.2015.136`
17. *Minaiev P.Ye., Pypka O.O.*: On Schur-type theorem for Leibniz 3-algebras. Bulletin of the Taras Shevchenko National University of Kyiv. Physics and Mathematics. 2024; 78(1): pp. 22–25. `doi:10.17721/1812-5409.2024/1.3`
18. *Minaiev P.Ye., Pypka O.O., Shyshenko I.V.*: On Poisson (2-3)-algebras which are finite-dimensional over the center. Res. Math. 2024; 32(1): pp. 118–133. `doi:10.15421/242411`
19. *Neumann B.H.*: Groups with finite classes of conjugate elements. P. London Math. Soc. 1951; 1(1): pp. 178–187. `doi:10.1112/plms/s3-1.1.178`

20. *Pypka A.A.*: Relationships between the factors of the central series and the nilpotent residual in some infinite groups. Adv. Group Theory Appl. 2017; 4: pp. 65–82. `doi:10.4399/97888255086975`
21. *Riley D., Shahada M.*: Relationships between the canonical ascending and descending central series of ideals of an associative algebra. Commun. Algebra 2017; 45(5): pp. 1969–1982. `doi:10.1080/00927872.2016.1226871`
22. *Stewart I.N.*: Verbal and marginal properties of non-associative algebras in the spirit of infinite group theory. P. London Math. Soc. 1974; 28(1): pp. 129–140. `doi:10.1112/plms/s3-28.1.129`
23. *Saeedi F., Veisi B.*: On Schur theorem and it converses for n-Lie algebras. Linear Multilinear Algebra 2014; 62(9): pp. 1139–1145. `doi:10.1080/03081087.2013.809871`
24. *Vaughan-Lee M.R.*: Metabelian BFC p-groups. J. London Math. Soc. 1972; s2-5(4): pp. 673–680. `doi:10.1112/jlms/s2-5.4.673`
25. *Ushakov V.I.*: Topological groups with bicompact classes of conjugate subgroups. Mat. Sb. 1964; 63(105): pp. 277–283.

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