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Transposed Poisson structures on $\mathfrak{osp}(1,2)$ [1]

Abstract. In this article, we describe transposed Poisson superalgebra structures on the 5-dimensional orthosymplectic Lie superalgebra $\mathfrak{osp}(1,2)$ in characteristic 3. Specifically, we demonstrate that on $\mathfrak{osp}(1,2)$, there is only trivial transposed Poisson structure.
Key words: transposed Poisson superalgebra, orthosymplectic Lie superalgebra, δ-derivation

Анотація. У цій статті ми описуємо структури транспонованої супералгебри Пуассона на 5-вимірній ортосимплектичній супералгебрі Лі $\mathfrak{osp}(1,2)$ у характеристиці 3. Зокрема, ми показуємо, що на $\mathfrak{osp}(1,2)$ існує лише тривіальна транспонована структура Пуассона.
Ключові слова: транспонована супералгебра Пуассона, ортосимплектична супералгебра Лі, δ-диференціювання

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1. Introduction

Poisson algebras have their roots in the Poisson geometry of the 1970s. Since then, Poisson algebras have emerged in various branches of physics and mathematics. Currently, the transposed Poisson algebra is a dual concept of the Poisson algebra that was recently introduced by Bai, Bai, Guo, and Wu [2]. Consequently, the transposed Poisson algebra and the transposed Poisson superalgebra on some (super)algebras were studied by many authors [3, 4, 6], [5, 10, 11, 13, 15, 16, 20].

In ([6], Corollary 9) it was proved that there are no non-trivial transposed Poisson algebra structures defined on a complex semisimple finite-dimensional Lie algebra. In particular, on $\mathfrak{sl}_n(\mathbb{C})$. Further, for the Schrödinger algebra $\mathfrak{s} = \mathfrak{sl}_2(\mathbb{C}) \ltimes \mathfrak{H}$ (semi-direct sum), where $\mathfrak{H}$ is the 3-dimensional Heisenberg (nilpotent Lie) algebra, there are no non-trivial transposed Poisson algebra structures defined on $\mathfrak{s}$ ([6], Corollary 10).

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The centrally extended conformal Galilei algebra $\widetilde{\mathfrak{g}}^{(l)}$ (for $0 < l \in \mathbb{N} - \{\frac{1}{2}\}$) consists of $\mathfrak{sl}_2(\mathbb{C})$ and copies of the Heisenberg algebra, that is, $\widetilde{\mathfrak{g}}^{(l)}$ is a Lie algebra extension of $\mathfrak{sl}_2(\mathbb{C})$. In ([11], Section 4) the transposed Poisson structures were studied on $\widetilde{\mathfrak{g}}^{(l)}$. Specifically, there are no non-trivial transposed Poisson structures on $\widetilde{\mathfrak{g}}^{(l)}$.

In [12], the transposed Poisson structure on perfect Lie algebras was studied. More precisely, let $\mathfrak{L}^m = \mathfrak{sl}_2(\mathbb{C}) \ltimes \mathfrak{r}$ be the Lie algebra for $m \geq 2$, where $\mathfrak{r}$ is a $(m+1)$-dimensional irreducible representation of $\mathfrak{sl}_2(\mathbb{C})$. Then there are no non-trivial transposed Poisson algebra structures defined on $\mathfrak{L}^m$.

The Lie algebras $\mathfrak{s}$, $\widetilde{\mathfrak{g}}^{(l)}$, and $\mathfrak{L}^m$ are Lie algebras extensions of $\mathfrak{sl}_2(\mathbb{C})$.

The following result ([18], Corollary 7) generalizes the well known fact that claims that a complex simple finite-dimensional Lie algebra does not admit a non-trivial transposed Poisson structure (see [6], Corollary 9).

Corollary 1. *Let $\mathfrak{g}$ be a prime Lie algebra over a field of characteristic $p \neq 2, 3$ endowed with a non-degenerate symmetric invariant bilinear form. Then $\mathfrak{g}$ does not admit a non-trivial transposed Poisson structure.*

Moreover, in ([18], Subsection 2.3) it was shown that there are non-trivial simple finite-dimensional transposed Poisson algebras over fields of positive characteristic. For this, consider the special linear Lie algebra $\mathfrak{sl}_n(\mathbb{F})$, where $\mathbb{F}$ is a field of characteristic $p > 2$. If $p > 3$, by Corollary 1 any transposed Poisson structure on $\mathfrak{sl}_n(\mathbb{F})$ with $p \nmid n$ is trivial.

Furthermore, in [18] the author proved that there are non-trivial simple finite-dimensional transposed Poisson algebras over fields of prime characteristic. Specifically, it was proved that the modular Witt algebra $\mathcal{W}_p$ ($\mathcal{W}_p$ is the Lie algebra of derivations of the truncated polynomial algebra in one variable $\mathcal{A}_p = \mathbb{F}[x]/(x^p)$) has non-trivial simple finite-dimensional transposed Poisson algebras.

Theorem 1. *([18], Theorem 20) Let $(\mathcal{W}_p, \circ, [\cdot,\cdot])$ be a transposed Poisson structure defined on the modular Witt algebra $\mathcal{W}_p$ with $p > 3$. Then the algebra $(\mathcal{W}_p, \circ)$ is a mutation of an algebra isomorphic to the algebra of truncated polynomials in one variable $\mathcal{A}_p$. In particular, this mutation can be trivial. Moreover, if it is non-trivial, it is not a Poisson algebra.*

Note that $\mathfrak{sl}_2(\mathbb{F})$ coincides with the modular Witt algebra $\mathcal{W}_3$, when the base field $\mathbb{F}$ has the characteristic $p = 3$. The following Theorem 2 was also considered in [17] (Remark 7).

Theorem 2. *([18], Theorem 22) Let $(\mathcal{W}_3, \circ, [\cdot,\cdot])$ be a transposed Poisson structure. Then the algebra $(\mathcal{W}_3, \circ)$ is a mutation of an algebra isomorphic to the algebra of truncated polynomials in one variable $\mathcal{A}_3$. In particular, this mutation can be trivial. If it is non-trivial, it is not a Poisson algebra.*

From the current results and Theorems 1, 2, we have the following problems.

Problem 1. For the Lie algebra $\mathfrak{sl}_2(\mathbb{C})$, are there Lie (super)algebras extensions $\mathfrak{g}$ of $\mathfrak{sl}_2(\mathbb{C})$ such that there are non-trivial transposed Poisson algebra structures on $\mathfrak{g}$?

Problem 2. Are valid the Theorems 1 and 2 for any Lie (super)algebra extensions of the algebra $(\mathcal{W}_p, \circ)$, when $\mathbb{F}$ has characteristic $p \geq 3$?

The problems 1 and 2 consider both finite-dimensional simple and non-simple Lie (super)algebras.

In this paper, we respond negatively to the problem 2, that is, the problem 2 is false for the case $\mathfrak{sl}_2(\mathbb{F}) = \mathcal{W}_3$, that is, we prove that the orthosymplectic Lie superalgebra $\mathfrak{osp}(1,2)$, contrary to $\mathfrak{sl}_2(\mathbb{F})$, does not have non-trivial transposed Poisson algebra structures. Specifically, we demonstrate in Theorem 3 that the transposed Poisson algebra structures on $\mathfrak{osp}(1,2)$ are trivial.

2. Preliminaries

2.1. Transposed Poisson superalgebras

The concept of a transposed Poisson algebra was first described in [2], and the transposed Poisson superalgebra is a superization of that idea.

Definition 1. Let $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ be a $\mathbf{Z}_2$-graded vector space equipped with two nonzero bilinear super-operations $\cdot$ and $[\cdot,\cdot]$. The triple $(\mathfrak{g}, \cdot, [\cdot,\cdot])$ is called a transposed Poisson superalgebra (TPS) if $(\mathfrak{g}, \cdot)$ is a supercommutative associative superalgebra and $(\mathfrak{g}, [\cdot,\cdot])$ is a Lie superalgebra that satisfies the following compatibility condition

$$2c \cdot [a,b] = [c \cdot a, b] + (-1)^{|a||c|}[a, c \cdot b], \quad a,b,c \in \mathfrak{g}_0 \cup \mathfrak{g}_1. \tag{2.1}$$

Definition 2. Let $(\mathfrak{g}, [\cdot,\cdot])$ be a Lie superalgebra. A transposed Poisson superalgebra structure in $(\mathfrak{g}, [\cdot,\cdot])$ is a supercommutative associative multiplication $\cdot$ in $\mathfrak{g}$ which makes $(\mathfrak{g}, \cdot, [\cdot,\cdot])$ a TPS.

Definition 3. Let $(\mathfrak{g}, [\cdot,\cdot])$ be a superalgebra and φ be a homogeneous linear map $\mathfrak{g} \to \mathfrak{g}$. Then φ is called a $\frac{1}{2}$-superderivation if it satisfies

$$\varphi([a,b]) = \frac{1}{2}\left([\varphi(a), b] + (-1)^{|\varphi||a|}[a, \varphi(b)]\right), \quad a,b \in \mathfrak{g}_0 \cup \mathfrak{g}_1. \tag{2.2}$$

Note that $\frac{1}{2}$-superderivations represent a specific instance of δ-superderivations and that these, in turn, are generalizations of δ-derivations introduced in [7].

We will use the notation

$$D_{\frac{1}{2}}(\mathfrak{g}) = D^0_{\frac{1}{2}}(\mathfrak{g}) \oplus D^1_{\frac{1}{2}}(\mathfrak{g})$$

for the space of $\frac{1}{2}$-superderivations of a superalgebra $\mathfrak{g}$.

Lemma 1. *Let $(\mathfrak{g},\cdot,[\cdot,\cdot])$ be a transposed Poisson superalgebra and $a \in \mathfrak{g}_0 \cup \mathfrak{g}_1$. Then the left multiplication L_a of $(\mathfrak{g},\cdot)$ is a $\frac{1}{2}$-superderivation of $(\mathfrak{g},[\cdot,\cdot])$ and $|L_a| = |a|$.*

Multiplication by a field element is the simplest example of a $\frac{1}{2}$-superderivation. Such $\frac{1}{2}$-superderivations will be called *trivial*.

3. Transposed Poisson structures on $\mathfrak{osp}(1,2)$

In the following, the base field $\mathbb{F}$ has the characteristic $p = 3$.

Let $\mathbf{B} = \mathbf{B}(4,2)$ be the Shestakov superalgebra which is an exceptional alternative superalgebra of characteristic 3 (see [21]), where $\mathbf{B}_0 = \mathbf{M}_2(\mathbb{F})$, and $\mathbf{B}_1 = \mathbb{F}\cdot m_1 \oplus \mathbb{F}\cdot m_2$ is the 2-dimensional irreducible Cayley bimodule over $\mathbf{B}_0$, that is, $\mathbf{B}_0$ acts on $\mathbf{B}_1$ as follows:

$$e_{ij}\cdot m_k = \delta_{ik} m_j,\ \ i,j,k \in \{1,2\},$$

$$m\cdot a = a^\star \cdot m,$$

where $a \in \mathbf{B}_0$, $m \in \mathbf{B}_1$, and $a \mapsto a^\star$ is the symplectic involution in $\mathbf{B}_0 = \mathbf{M}_2(\mathbb{F})$. The odd multiplication in $\mathbf{B}_1$ is defined by

$$m_1^2 = -e_{21},\ m_2^2 = e_{12},\ m_1 m_2 = e_{11},\ \text{ and }\ m_2 m_1 = -e_{22}.$$

In characteristic $p = 3$, the orthosymplectic Lie superalgebra $\mathfrak{osp}(1,2)$ appears to be related to $\mathbf{B}(4,2)$. Specifically, the 5-dimensional simple Lie superalgebra $\mathfrak{osp}(1,2)$ is isomorphic to the quotient $\mathbf{B}(4,2)^{(-)}/\mathbb{F}\cdot I$, where I is the matrix identity of order 2×2 and $\mathbf{B}(4,2)^{(-)}$ is the commutator superalgebra of the Shestakov superalgebra $\mathbf{B}(4,2)$.

Now, since $\mathbb{F}$ has the characteristic 3, we will frequently use $2 = -1$ or $-2 = 1$. Thus, the Lie superalgebra $\mathfrak{osp}(1,2) = \mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ has the following structure: Let $\mathfrak{g}_0 = \mathfrak{sl}_2(\mathbb{F})$ be the 3-dimensional Lie algebra with the following basis $\{E,H,F\}$ with multiplication

$$EH = E,\ \ FH = -F,\ \ EF = \frac{1}{2}H \tag{3.1}$$

and let $\mathfrak{g}_1 = V_2 = \mathbb{F}\cdot u + \mathbb{F}\cdot v$ be the Lie module with basis $\{u,v\}$ such that

$$uH = u,\ vH = -v,\ uE = v,\ uF = 0,\ vE = 0,\ vF = -u. \tag{3.2}$$

Odd multiplication in $\mathfrak{g}_1$ is defined by

$$u^2 = F,\ \ uv = vu = \frac{1}{2}H,\ \text{and}\ v^2 = E. \tag{3.3}$$

Here we are explicitly using the notation $[\cdot,\cdot]$ for the supermultiplication in $\mathfrak{g}$.

The superalgebra $\mathfrak{osp}(1,2)$ is a Lie superalgebra extension of $\mathfrak{sl}_2(\mathbb{F})$ because $\mathfrak{sl}_2(\mathbb{F})$ is the even part of $\mathfrak{osp}(1,2)$.

Remark 1. If $p \neq 3$ we have the 5-dimensional superalgebra $\mathfrak{g}(3,2) = \mathfrak{sl}_2(\mathbb{F}) \oplus V_2$ with multiplication defined by (3.1), (3.2) and (3.3). In addition, the superalgebra $\mathfrak{g}(3,2)$ is also isomorphic to the quotient $B(4,2)^{(-)}/\mathbb{F} \cdot I$.

In ([14], Section 8) it was proved that the simple superalgebra $\mathfrak{g}(3,2)$ is not a binary-Lie superalgebra; thus, $\mathfrak{g}(3,2)$ is not a Lie superalgebra.

3.1. Even $\frac{1}{2}$-derivations of $\mathfrak{osp}(1,2)$

In this subsection, we only take into account *even* linear maps $\varphi : \mathfrak{osp}(1,2) \to \mathfrak{osp}(1,2)$, that is, those that satisfy $\varphi(\mathfrak{g}_i) \subseteq \mathfrak{g}_i$ for $i \in \{0,1\}$. Thus $|\varphi| = 0$, then φ is a $\frac{1}{2}$-superderivation of $\mathfrak{osp}(1,2)$ if and only if φ is a usual $\frac{1}{2}$-derivation of $\mathfrak{osp}(1,2)$. Let D be an even $\frac{1}{2}$-superderivation of $\mathfrak{g} = \mathfrak{osp}(1,2)$, then from (2.2) we have

$$D(ab) = \frac{1}{2}(D(a)b + aD(b)) \tag{3.4}$$

for all homogeneous elements $a, b \in \mathfrak{g}_0 \cup \mathfrak{g}_1$.

Consider

$$D(E) = \alpha_{11}E + \alpha_{12}H + \alpha_{13}F, \tag{3.5}$$
$$D(H) = \alpha_{21}E + \alpha_{22}H + \alpha_{23}F, \tag{3.6}$$
$$D(F) = \alpha_{31}E + \alpha_{32}H + \alpha_{33}F, \tag{3.7}$$
$$D(u) = \beta_{11}u + \beta_{12}v, \tag{3.8}$$
$$D(v) = \beta_{21}u + \beta_{22}v, \tag{3.9}$$

where $\alpha_{ij}, \beta_{ij} \in \mathbb{F}$. In the sequel, we will frequently use the relation (3.4); so, from (3.1) we get

$$D(E) = D(EH) = -(D(E)H + ED(H)),$$

$$\begin{aligned}\alpha_{11}E + \alpha_{12}H + \alpha_{13}F &\\ &= -(\alpha_{11}E + \alpha_{12}H + \alpha_{13}F)H - E(\alpha_{21}E + \alpha_{22}H + \alpha_{23}F)\\ &= -\alpha_{11}E + \alpha_{13}F - \alpha_{22}E - \frac{1}{2}\alpha_{23}H,\end{aligned}$$

then $\alpha_{22} = \alpha_{11}$ and $\alpha_{23} = \alpha_{12}$. Thus

$$D(H) = \alpha_{21}E + \alpha_{11}H + \alpha_{12}F, \tag{3.10}$$

Also,

$$-D(F) \overset{(3.1)}{=} D(FH) = -(D(F)H + FD(H)),$$

and by (3.7) and (3.10)

$$\begin{aligned}-\alpha_{31}E - \alpha_{32}H - \alpha_{33}F &\\ &= -(\alpha_{31}E + \alpha_{32}H + \alpha_{33}F)H - F(\alpha_{21}E + \alpha_{11}H + \alpha_{12}F)\\ &= -\alpha_{31}E + \alpha_{33}F + \frac{1}{2}\alpha_{21}H + \alpha_{11}F,\end{aligned}$$

then $\alpha_{33} = \alpha_{11}$ and $\alpha_{32} = \alpha_{21}$. Hence

$$D(F) = \alpha_{31}E + \alpha_{21}H + \alpha_{11}F. \tag{3.11}$$

Further, by

$$\frac{1}{2}D(H) \overset{(3.1)}{=} D(EF) = -(D(E)F + ED(F)),$$

$$\begin{aligned} -\alpha_{21}E - \alpha_{11}H - \alpha_{13}F & \\ &= -(\alpha_{11}E + \alpha_{12}H + \alpha_{13}F)F - E(\alpha_{31}E + \alpha_{21}H + \alpha_{11}F) \\ &= -\frac{1}{2}\alpha_{11}H - \alpha_{12}F - \alpha_{21}E - \frac{1}{2}\alpha_{11}H, \end{aligned}$$

and the equality does not offer any information.

In addition,

$$D(u) \overset{(3.2)}{=} D(uH) = -(D(u)H + uD(H)),$$

$$\begin{aligned} \beta_{11}u + \beta_{12}v &\overset{(3.8),(3.10)}{=} -(\beta_{11}u + \beta_{12}v)H - u(\alpha_{21}E + \alpha_{11}H + \alpha_{12}F) \\ &= -\beta_{11}u + \beta_{12}v - \alpha_{21}v - \alpha_{11}u \end{aligned}$$

subsequently $\beta_{11} = \alpha_{11}$ and $\alpha_{21} = 0$. So, from (3.10), (3.11) and (3.8) we have

$$D(H) = \alpha_{11}H + \alpha_{12}F, \tag{3.12}$$

$$D(F) = \alpha_{31}E + \alpha_{11}F, \tag{3.13}$$

$$D(u) = \alpha_{11}u + \beta_{12}v. \tag{3.14}$$

On the other hand, from

$$0 \overset{(3.2)}{=} D(uF) = -(D(u)F + uD(F)),$$

we have

$$0 \overset{(3.14),(3.13)}{=} -(\alpha_{11}u + \beta_{12}v)F - u(\alpha_{31}E + \alpha_{11}F) = \beta_{12}u - \alpha_{31}v$$

and $\beta_{12} = \alpha_{31} = 0$. Therefore, substituting in (3.13) and (3.14) we get

$$D(F) = \alpha_{11}F, \tag{3.15}$$

$$D(u) = \alpha_{11}u. \tag{3.16}$$

Moreover,

$$-D(v) \overset{(3.2)}{=} D(vH) = -(D(v)H + vD(H)),$$

$$\begin{aligned} -\beta_{21}u - \beta_{22}v &\overset{(3.9),(3.12)}{=} -(\beta_{21}u + \beta_{22}v)H - v(\alpha_{11}H + \alpha_{12}F) \\ &= -\beta_{21}u + \beta_{22}v + \alpha_{11}v + \alpha_{12}u \end{aligned}$$

then $\beta_{22} = \alpha_{11}$ and $\alpha_{12} = 0$. Therefore, from (3.5), (3.12) and (3.9) we get

$$D(E) = \alpha_{11}E + \alpha_{13}F, \tag{3.17}$$
$$D(H) = \alpha_{11}H, \tag{3.18}$$
$$D(v) = \beta_{21}u + \alpha_{11}v. \tag{3.19}$$

But from

$$0 \overset{(3.2)}{=} D(vE) = -(D(v)E + vD(E)),$$
$$0 \overset{(3.19),(3.17)}{=} -(\beta_{21}u + \alpha_{11}v)E - v(\alpha_{11}E + \alpha_{13}F) = -\beta_{21}v + \alpha_{13}u$$

then $\alpha_{13} = 0$ and $\beta_{21} = 0$. Thus, from (3.17) and (3.19)

$$D(E) = \alpha_{11}E, \tag{3.20}$$
$$D(v) = \alpha_{11}v. \tag{3.21}$$

Furthermore, from

$$-D(u) \overset{(3.2)}{=} D(vF) = -(D(v)F + vD(F)),$$
$$-\alpha_{11}u = -(\alpha_{11}v)F - v(\alpha_{11}F) = \alpha_{11}u + \alpha_{11}u = 2\alpha_{11}u,$$

and the equality does not offer any information. Also, from

$$D(v) \overset{(3.2)}{=} D(uE) = -(D(u)E + uD(E)),$$

we get

$$\alpha_{11}v \overset{(3.16),(3.20)}{=} -(\alpha_{11}u)E - u(\alpha_{11}E) = -\alpha_{11}v - \alpha_{11}v = -2\alpha_{11}v,$$

and the equality does not offer any information. Similarly, from (3.3) and (3.4) it is easy to see that the relations

$$D(E) = D(v^2) = -D(v)v - vD(v),$$
$$-D(H) = D(uv) = -D(u)v - uD(v),$$
$$D(F) = D(u^2) = -D(u)u - u(D(u)),$$

does not offer any information. Thus, summarizing from (3.20), (3.18), (3.15), (3.16) and (3.21) we have the following

$$D(E) = \alpha_{11}E,$$
$$D(H) = \alpha_{11}H,$$
$$D(F) = \alpha_{11}F,$$
$$D(u) = \alpha_{11}u,$$
$$D(v) = \alpha_{11}v.$$

Then, $D(x) = \alpha x$ for any $x \in \mathfrak{osp}(1,2)$ and some $\alpha \in \mathbb{F}$. Also the matrix associated with the even $\frac{1}{2}$-superderivation D is

$$[D] = \begin{pmatrix} \alpha & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & \alpha \end{pmatrix} = \alpha I$$

for some $\alpha \in \mathbb{F}$, where I is the matrix identity.

Proposition 1. We have $D^0_{\frac{1}{2}}(\mathfrak{g}) = \langle \mathrm{id} \rangle$, where id is the identity map.

3.2. Odd $\frac{1}{2}$-derivations of $\mathfrak{osp}(1,2)$

This subsection solely takes into account *odd* linear maps $\varphi : \mathfrak{osp}(1,2) \to \mathfrak{osp}(1,2)$, i.e. those which satisfy $\varphi(\mathfrak{g}_i) \subseteq \mathfrak{g}_{1-i}$ for $i \in \{0,1\}$. Thus $|\varphi| = 1$, and from (2.2), D is an odd $\frac{1}{2}$-superderivation of $\mathfrak{g} = \mathfrak{osp}(1,2)$ if and only if

$$D(ab) = \frac{1}{2}(D(a)b + (-1)^{|a|}aD(b)) \tag{3.22}$$

for all $a, b \in \mathfrak{g}_0 \cup \mathfrak{g}_1$.

Consider

$$D(E) = \alpha_{11}u + \alpha_{12}v, \tag{3.23}$$
$$D(H) = \alpha_{21}u + \alpha_{22}v, \tag{3.24}$$
$$D(F) = \alpha_{31}u + \alpha_{32}v, \tag{3.25}$$
$$D(u) = \alpha_{41}E + \alpha_{42}H + \alpha_{43}F, \tag{3.26}$$
$$D(v) = \alpha_{51}E + \alpha_{52}H + \alpha_{53}F. \tag{3.27}$$

where $\alpha_{ij} \in \mathbb{F}$. Here we will frequently use the relation (3.22); then, from (3.1) and (3.23) we have

$$D(E) = D(EH) = -(D(E)H + ED(H)),$$

$$\alpha_{11}u + \alpha_{12}v = -(\alpha_{11}u + \alpha_{12}v)H - E(\alpha_{21}u + \alpha_{22}v) = -\alpha_{11}u + \alpha_{12}v + \alpha_{21}v,$$

then $\alpha_{11} = \alpha_{21} = 0$; so

$$D(E) = \alpha_{12}v, \tag{3.28}$$
$$D(H) = \alpha_{22}v. \tag{3.29}$$

Also

$$-D(F) \overset{(3.1)}{=} D(FH) = -(D(F)H + FD(H)),$$

imply

$$-\alpha_{31}u - \alpha_{32}v \overset{(3.25),(3.29)}{=} -(\alpha_{31}u + \alpha_{32}v)H - F(\alpha_{22}v) = -\alpha_{31}u + \alpha_{32}v - \alpha_{22}u,$$

then $\alpha_{32} = \alpha_{22} = 0$, then from (3.29) and (3.25) we get

$$D(H) = 0, \tag{3.30}$$
$$D(F) = \alpha_{31}u. \tag{3.31}$$

Further, by

$$\frac{1}{2}D(H) \overset{(3.1)}{=} D(EF) = -(D(E)F + ED(F)),$$

$0 = -(\alpha_{12}v)F - E(\alpha_{31}u) = \alpha_{12}u + \alpha_{31}v$, then $\alpha_{12} = \alpha_{31} = 0$. Thus, from (3.28) and (3.31) we have

$$D(E) = D(F) = 0. \tag{3.32}$$

Besides, from

$$D(u) \overset{(3.2)}{=} D(uH) = -(D(u)H - uD(H)),$$

$$\alpha_{41}E + \alpha_{42}H + \alpha_{43}F \overset{(3.26),(3.30)}{=} -(\alpha_{41}E + \alpha_{42}H + \alpha_{43}F)H = -\alpha_{41}E + \alpha_{43}F$$

then $\alpha_{41} = \alpha_{42} = 0$. So, from (3.26) we have

$$D(u) = \alpha_{43}F. \tag{3.33}$$

On the other hand,

$$0 \overset{(3.2)}{=} D(uF) = -(D(u)F - uD(F)),$$

imply

$$0 \overset{(3.33),(3.32)}{=} -(\alpha_{43}F)F,$$

and the equality does not offer any information. Moreover, from (3.2)

$$-D(v) = D(vH) = -(D(v)H - vD(H)),$$

$$-\alpha_{51}E - \alpha_{52}H - \alpha_{53}F \overset{(3.27),(3.30)}{=} -(\alpha_{51}E + \alpha_{52}H + \alpha_{53}F)H = -\alpha_{51}E + \alpha_{53}F$$

then $\alpha_{52} = \alpha_{53} = 0$. Therefore, from (3.27) we get

$$D(v) = \alpha_{51}E. \tag{3.34}$$

Also,

$$0 \overset{(3.2)}{=} D(vE) = -(D(v)E - vD(E)),$$
$$0 \overset{(3.34),(3.32)}{=} -(\alpha_{51}E)E,$$

and the equality does not offer any information. Furthermore,

$$-D(u) \overset{(3.2)}{=} D(vF) = -(D(v)F - vD(F)),$$

$$-\alpha_{43}F = -(\alpha_{51}E)F = \alpha_{51}H,$$

then $\alpha_{43} = \alpha_{51} = 0$. So

$$D(u) = D(v) = 0. \tag{3.35}$$

Thus, from (3.30), (3.32) and (3.35), we conclude that $D = 0$.

Proposition 2. $D^1_{\frac{1}{2}}(\mathfrak{g}) = \{0\}$.

Finally, we now describe the transposed Poisson superalgebra structures on $(\mathfrak{g}, [\cdot,\cdot])$, where $\mathfrak{g} = \mathfrak{osp}(1,2)$.

Theorem 3. *All transposed Poisson superalgebra structures on* $(\mathfrak{g}, [\cdot,\cdot])$ *are trivial.*

Proof. Let $(\mathfrak{g}, \cdot, [\cdot,\cdot])$ be a TPS, that is, $(\mathfrak{g}, \cdot)$ is supercommutative and (2.1) hold. Denote by φ_a and ψ_c the left multiplications by $a \in \mathfrak{g}_0$ and $c \in \mathfrak{g}_1$, respectively, in $(\mathfrak{g}, \cdot)$.

Let $a, b \in \mathfrak{g}_0$ and $c, d \in \mathfrak{g}_1$

$$b \cdot a = \varphi_b(a), \quad a \cdot c = \varphi_a(c), \tag{3.36}$$

$$c \cdot a = \psi_c(a), \quad c \cdot d = \psi_c(d), \tag{3.37}$$

In view of the supercommutativity of $(\mathfrak{g}, \cdot)$, we have

$$b \cdot a = \varphi_a(b), \quad a \cdot c = \psi_c(a), \tag{3.38}$$

$$c \cdot a = \varphi_a(c), \quad c \cdot d = -\psi_d(c), \tag{3.39}$$

By (2.1) we have $\varphi_E, \varphi_H, \varphi_F \in D^0_{\frac{1}{2}}(\mathfrak{g})$ and $\psi_u, \psi_v \in D^1_{\frac{1}{2}}(\mathfrak{g})$. Then

$$\varphi_E = \alpha_E \mathrm{id}, \ \ \varphi_H = \alpha_H \mathrm{id} \ \text{ and } \ \varphi_F = \alpha_F \mathrm{id}$$

for some $\alpha_E, \alpha_H, \alpha_F \in \mathbb{F}$ by Proposition 1 and $\psi_u = \psi_v = 0$ by Proposition 2. It follows from (3.36) and (3.38) that $\alpha_E = \alpha_H = \alpha_F = 0$. Thus, $\cdot$ is trivial.

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References

1. *Albuquerque H., Barreiro E., Benayadi S., Boucetta M., Sánchez J. M.*: Poisson algebras and symmetric Leibniz bialgebra structures on oscillator Lie algebras. Journal of Geometry and Physics 2021; 160: 103939. doi:10.1016/j.geomphys.2020.103939

2. *Bai C., Bai R., Guo L., Wu Y.*: Transposed Poisson algebras, Novikov-Poisson algebras and 3-Lie algebras. Journal of Algebra 2023; 632: pp. 535–566. doi:10.1016/j.jalgebra.2023.06.006
3. *D. Beites P., Ferreira B. L. M., Kaygorodov I.*: Transposed Poisson structures. Results in Mathematics 2024; 79(2): 93. doi:10.1007/s00025-023-02107-x
4. *Billig Y.*: Towards Kac-van de Leur Conjecture: Locality of superconformal algebras. Advances in Mathematics 2022; 400: 108295. doi:10.1016/j.aim.2022.108295
5. *Fehlberg Júnior R., Kaygorodov I.*: On the Kantor Product, II. Carpathian Math. Publ. 2022; 14: pp. 543–563. doi:10.15330/cmp.14.2.543-563
6. *Ferreira B. L. M., Kaygorodov I., Lopatkin V.*: $\frac{1}{2}$-derivations of Lie algebras and transposed Poisson algebras. Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas 2021; 115(3): 142. doi:10.1007/s13398-021-01088-2
7. *Filippov V. T. E.*: On δ-derivations of Lie algebras. Siberian Mathematical Journal 1998; 39(6): pp. 1218–1230. doi:10.1007/BF02674132
8. *Jaworska-Pastuszak A., Pogorzały Z.*: Poisson structures for canonical algebras. Journal of Geometry and Physics 2020; 148: 103564. doi:10.1016/j.geomphys.2019.103564
9. *Kaygorodov I., Khrypchenko M.*: Poisson structures on finitary incidence algebras. Journal of Algebra 2021; 578: pp. 402–420. doi:10.1016/j.jalgebra.2021.03.011
10. *Kaygorodov I., Khrypchenko M.*: Transposed Poisson structures on Block Lie algebras and superalgebras. Linear Algebra and its Applications 2023; 656: pp. 167–197. doi:10.1016/j.laa.2022.09.024
11. *Kaygorodov I., Lopatkin V., Zhang Z.*: Transposed Poisson structures on Galilean and solvable Lie algebras. Journal of Geometry and Physics 2023; 187: 104781. doi:10.1016/j.geomphys.2023.104781
12. *Kaygorodov I., Khudoyberdiyev A.*: Transposed Poisson structures on solvable and perfect Lie algebras. J. Phys. A: Math. Theor. 2024; 57: 035205. doi:10.1088/1751-8121/ad1620
13. *Laraiedh I., Silvestrov S.*: Transposed Hom-Poisson and Hom-pre-Lie Poisson algebras. Hacettepe Journal of Mathematics and Statistics 2026; 55(2): pp. 473–501. doi:10.15672/hujms.1417353
14. *López Solís V.*: Malcev $\mathfrak{sl}_2$-algebras. Journal of Algebra and Its Applications 2025; 2650273. doi:10.1142/S0219498826502737
15. *Ma T., Li B.*: Transposed BiHom-Poisson algebras. Communications in Algebra 2023; 51(2): pp. 528–551. doi:10.1080/00927872.2022.2105343
16. *Martínez C., Zelmanov E.*: Brackets, superalgebras and spectral gap. São Paulo Journal of Mathematical Sciences 2019; 13(1): pp. 112–132. doi:10.1007/s40863-019-00128-5
17. *Ouaridi A. F.*: On the simple transposed Poisson algebras and Jordan superalgebras. arXiv preprint arXiv:2305.13848, 2023.
18. *Ouaridi A. F.*: On the simple transposed Poisson algebras and Jordan superalgebras. Journal of Algebra 2024; 641: pp. 173–198. doi:10.1016/j.jalgebra.2023.11.026
19. *Yao Y., Ye Y., Zhang P.*: Quiver Poisson algebras. Journal of Algebra 2007; 312(2): pp. 570–589. doi:10.1016/j.jalgebra.2007.03.034

20. *Yuan L., Hua Q.*: $\frac{1}{2}$-(bi)derivations and transposed Poisson algebra structures on Lie algebras. Linear and Multilinear Algebra 2022; 70(22): pp. 7672–7701. doi:10.1080/03081087.2021.2003287
21. *Shestakov I. P.*: Prime alternative superalgebras of arbitrary characteristic. Algebra and logic 1997; 36(6): pp. 389–412. doi:10.1007/BF02671556

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